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M. M. Weiner

Influence of Non-Homogeneous Earth on the Performance of High-Frequency Receiving Arrays with Electrically-Small Ground Planes

MITRE
Bedford, Massachusetts

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ABSTRACT

The performance of ground-based high-frequency (HF) antenna arrays is reduced when the array elements have electrically-small ground planes. Performance degradations include: (1) a decrease in directive gain near the horizon (caused by earth multipath), (2) a decrease in radiation efficiency and an increase in internal noise (caused by ground losses), (3) an array RMS phase error (caused by exterior currents on element feed cables), and (4) an array rms phase error and beam pointing errors (caused by non-uniform Fresnel reflection RMS from a non-homogeneous earth).

This paper models the degradation described in (4) above. Numerical results are presented for cases of randomly-distributed and systematically-distributed earth non-homogeneities where one-half of vertically-polarized array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. The maximum rms phase errors, for the cases examined, are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum beam pointing errors are 0 and 0.3 beamwidths for randomly-distributed and systematically-distributed non-homogeneities, respectively.

ACKNOWLEDGMENT

The least square's algorithm given by equation (A-3) in the appendix was brought to the author's attention by J. D. R. Kramer.

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SECTION 1

INTRODUCTION

High-frequency (HF) over-the-horizon (OTH) radars generally employ transmitting and receiving antenna arrays whose elements are in close proximity to earth, but with electrically-large ground planes to mitigate the influence of earth on antenna performance. However, in the design of advanced OTH radar systems with sparse receiving arrays comprising hundreds or thousands of elements pseudo-randomly distributed over a large area, electrically-large metallic ground planes are prohibitively expensive to construct or to maintain for so many elements. One alternative approach is to employ salt-water ground planes to achieve an electrically-large ground plane. However, suitable salt-water sites are not readily available and have their own unique problems. A second alternative approach is to use elements with electrically-small ground planes and to accept the reduction in antenna performance caused by a greater susceptibility to the influence of the earth. This second alternative approach is considered in this paper.

The performance of ground-based HF antenna arrays is reduced when the array elements have electrically-small ground planes. Performance degradations include (1) a decrease in directive gain near the horizon (caused by earth multipath), (2) a decrease in radiation efficiency and an increase in internal noise (caused by ground losses), (3) an array RMS phase error (caused by field-induced exterior currents on element feed cables), and (4) an array RMS phase error and beam pointing errors (caused by non-uniform Fresnel reflection from a non-homogeneous earth).

This paper models the array RMS phase error and beam pointing errors caused by non-uniform Fresnel reflection from a non-homogeneous earth. Numerical results are presented for cases of randomly-distributed and systematically-distributed earth non-homogeneities, where one-half of vertically-polarized array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. It is found, for the cases examined, that the maximum expected values of

RMS phase error are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum expected values of the beam-pointing error are 0 and 0.3 beamwidths from randomly-distributed and systematically-distributed non-homogeneities, respectively.

The performance degradation, caused by non-uniform Fresnel reflection by a non-homogeneous earth is modeled in section 2. Numerical results are presented in section 3. The summary and conclusions are given in section 4.

SECTION 2

MODEL

Consider a sparse HF receiving array of m vertically-polarized elements with ground planes, of radius a, pseudo-randomly distributed on flat earth over a circular area of radius $r_A >> a$. The elements, in close proximity to earth, are identical except that the earth below each element may vary from element to element. The element length ℓ and ground plane radius a are assumed to be electrically-small ($\ell \ll \lambda_o$, $a \ll \lambda_o$ where λ_o is the RF wavelength in free space). The midpoint of each element is at a height h above the earth.

Consider now a plane wave incident from the true direction (θ, Φ) where θ is the elevation angle of incident with respect to zenith (the z axis) and Φ is the azimuthal angle with respect to the x axis. The electric field at the midpoint of the k-th element, in the absence of mutual coupling among elements, is the sum of the fields from a direct ray incident from the direction (θ, Φ) and an indirect ("multipath") ray reflected at a point P_k at a horizontal distance h tan θ from the element local origin O_k (see figure 1). We assume that the ground plane is sufficiently small so that the indirect ray is reflected from the earth rather than from the ground plane. Accordingly, the radius of the ground plane satisfies the condition $a < h \tan \theta$. (At angles of incidence $\theta < 60°$ this condition is more stringent than the condition $a << \lambda_o$.)

The radiation pattern of the kth element is then approximately identical to that of a vertically-polarized Hertzian dipole at height h above flat earth (see figure 1).

The relative permittivity $\varepsilon_k^* / \varepsilon_o$ of the earth at the kth element is given by

$$\varepsilon_k^* / \varepsilon_o = \varepsilon_{rk} - j\, 60\, \lambda_o\, \sigma_k \tag{2-1}$$

where

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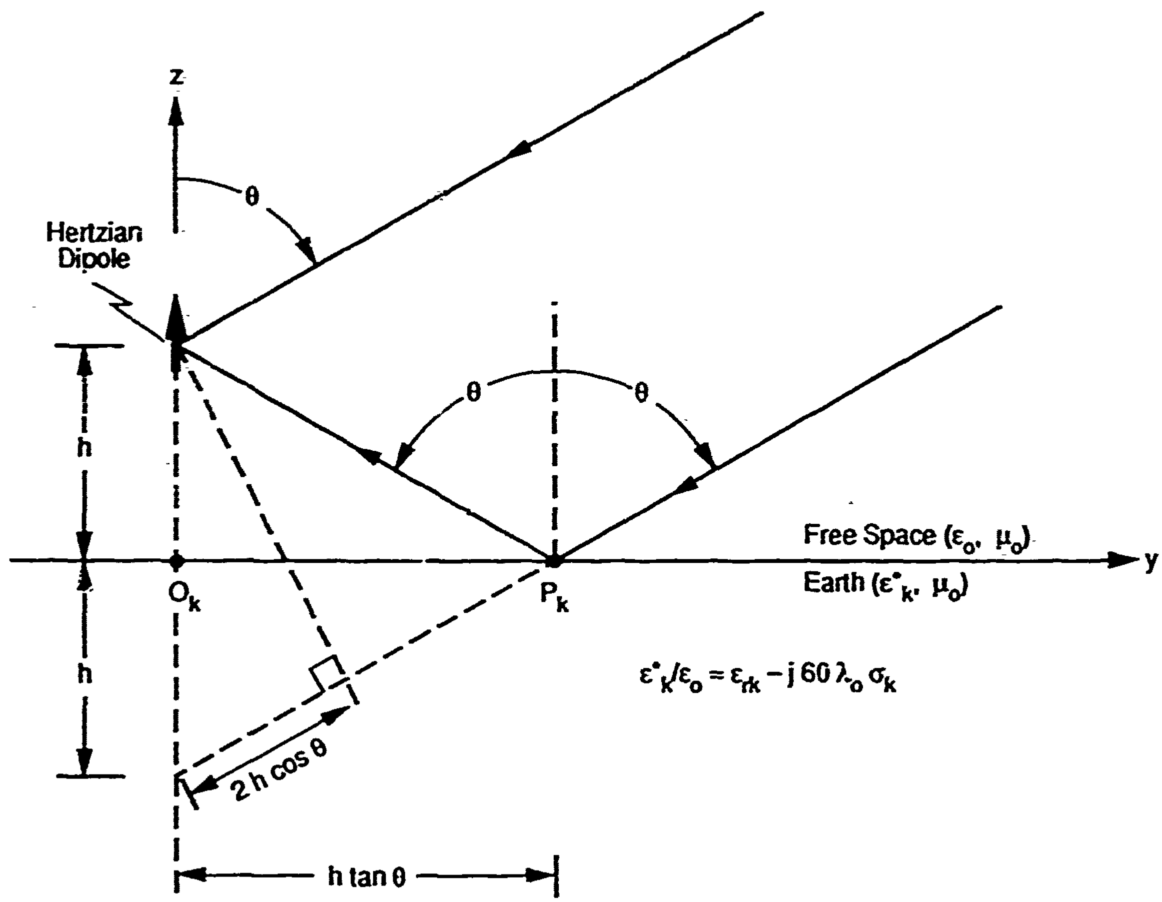

Figure 1. The kth Element Approximated by a Vertically-Polarized Hertzian Dipole Above Flat Earth

ε_{rk} = dielectric constant of the earth at the kth element (numeric)

σ_k = conductivity of the earth at the kth element (S/m)

λ_o = RF wavelength in free space (m)

For homogeneous earth,

$$\varepsilon^* / \varepsilon_o = \varepsilon_r - j\, 60\, \lambda_o\, \sigma \tag{2-2}$$

The ground constants ε_r and σ, loss tangent, and penetration depth δ, for the International Radio Consulative Committee (CCIR) 527-1 classifications of homogeneous earth, are summarized in table 1.

The field E_k (θ,ϕ) at the kth element, after elimination of the true phase advance of the direct field at each element relative to that at the origin, is given by

$$E_k(\theta) = A \sin\theta \left\{1 + \mathrm{R}_{k,\mathrm{v}} \exp[-j(2\pi / \lambda_o)2h \cos\theta]\right\} \quad 0 \le \theta \le \pi/2 \text{ rad} \tag{2-3}$$

where

A = constant (V/m)
θ = angle of incidence (measured from zenith) of the direct ray (degrees)
$\mathrm{R}_{k,v} = |\mathrm{R}_{k,v}| \exp[j(\text{angle } \mathrm{R}_{k,v})]$ = Fresnel reflection coefficient of the earth at the kth element for a vertically-polarized direct wave incident from the direction (θ,ϕ).

Equation (2-3) is valid for computing the indirect field in air relative to the direct field in air. The single ray (plane wave) Fresnel reflection coefficient model of equation (2-3) is not

Table 1. Permittivity, Loss Tangent, and Penetration Depth of CCIR-527-1 Classifications of Earth

Cases	CONSTANTS		LOSS TANGENT			PENETRATION DEPTH		
	ε_r	σ (S/m)	$\sigma/(\omega\varepsilon_r\varepsilon_o) = (60\lambda)(\sigma/\varepsilon_r)$			δ (m)		
			Frequency (MHz) [Wavelength (m)]			Frequency (MHz) [Wavelength (m)]		
			3 [99.93]	15 [19.986]	30 [9.993]	3 [99.93]	15 [19.986]	30 [9.993]
(1) Perfect Ground	1.0	∞	∞	∞	∞	0	0	0
(2) Sea Water (average Salinity 20°C)	70.0	5.0	4.282×10^{2}	8.425×10^{1}	4.283×10^{1}	1.3×10^{-1}	5.8×10^{-2}	4.1×10^{-2}
(3) Fresh Water	80.0	3.0×10^{-2}	2.251×10^{0}	4.497×10^{-1}	2.248×10^{-1}	2.1×10^{0}	1.6×10^{0}	1.6×10^{0}
(4) Wet Ground	30.0	1.0×10^{-2}	1.999×10^{0}	3.997×10^{-1}	1.999×10^{-1}	3.7×10^{0}	3.0×10^{0}	2.9×10^{0}
(5) Medium Dry Ground	15.0	1.0×10^{-3}	3.997×10^{-1}	7.995×10^{-2}	3.997×10^{-2}	2.1×10^{1}	2.1×10^{1}	2.1×10^{1}
(6) Very Dry Ground	3.0	1.0×10^{-4}	1.999×10^{-1}	3.997×10^{-2}	1.999×10^{-2}	9.2×10^{1}	9.2×10^{1}	9.2×10^{1}
(7) Pure Water, 20°C	80.0	1.8×10^{-6}	1.350×10^{-4}	-	-	2.6×10^{4}	-	-
		5.0×10^{-4}	-	7.495×10^{-3}	-	-	9.4×10^{2}	-
		1.7×10^{-3}	-	-	1.274×10^{-2}	-	-	2.7×10^{1}
(8) Ice (fresh water, -1°C)	3.0	6.0×10^{-5}	1.199×10^{-1}	-	-	1.5×10^{2}	-	-
		9.0×10^{-5}	-	3.597×10^{-2}	-	-	1.0×10^{2}	-
		1.0×10^{-4}	-	-	1.999×10^{-2}	-	-	9.2×10^{1}
(9) Ice (fresh water, -10°C)	3.0	1.8×10^{-5}	3.597×10^{-2}	-	-	5.1×10^{2}	-	-
		2.7×10^{-5}	-	1.079×10^{-2}	-	-	3.4×10^{2}	-
		3.5×10^{-5}	-	-	6.995×10^{-3}	-	-	2.6×10^{2}
(10) Average Land (TCI)	10.0	5.0×10^{-3}	2.998×10^{0}	5.996×10^{-1}	2.998×10^{-1}	4.8×10^{0}	1.6×10^{0}	3.4×10^{0}
(11) Free Space	1.0	0	0	0	0	∞	∞	∞

valid for calculating the amplitude A of the direct wave because the model does not include the substantial surface wave near-field losses in the earth.

The Fresnel reflection $R_{k,v}$ for parallel (vertical) polarization (the E-field is parallel to the plane of incidence) is given by [1]

$$R_{k,v} = \frac{(\varepsilon_k^* / \varepsilon_o)\cos\theta \; - [(\varepsilon_k^* / \varepsilon_o) - \sin^2\theta]^{1/2}}{(\varepsilon_k^* / \varepsilon_o)\cos\theta \; + [(\varepsilon_k^* / \varepsilon_o) - \sin^2\theta]^{1/2}} \tag{2-4}$$

where ε_k^* is given by equation (2-1). The Fresnel reflection coefficient $R_{k,v}$ is a function of k and θ. For homogeneous earth, equation (2-4) reduces to

$$R_{k,v} = R_v = \frac{(\varepsilon^* / \varepsilon_o)\cos\theta \; - [(\varepsilon^* / \varepsilon_o) - \sin^2\theta]^{1/2}}{(\varepsilon^* / \varepsilon_o)\cos\theta \; + [(\varepsilon^* / \varepsilon_o) - \sin^2\theta]^{1/2}}, \text{homogeneous earth} \tag{2-5}$$

where $\varepsilon_k^* / \varepsilon_o$ is given by equation (2-2). For homogeneous earth, R_v is a function only of the angle of incidence θ and the relative permittivity $\varepsilon^* / \varepsilon_o$. The Fresnel reflection coefficient R_v, computed by MITRE program MODIFIED IMAGES, is given in tables 2 through 11 at 6, 15, and 30 MHz for the ground constants specified in table 1. The coefficient R_v is tabulated as a function of the grazing angle ψ (with respect to zenith). Please note that $\psi = (\pi/2) - \theta$ (rad) or $\psi = 90 - \theta$ (degree).

The numeric directive gain $d(\theta)$ of a vertically-polarized Hertzian dipole at a height h above flat homogeneous <u>lossy</u> earth is given by reference 2 as

$$d(\theta) = \begin{cases} \dfrac{2|E(\theta)|^2}{\int_o^{\pi/2} |E(\theta)|^2 \sin\theta \, d\theta}, & 0 \le \theta \le \pi/2 \\ & \text{lossy earth} \\ 0 & , \ -\pi/2 \le \theta \le 0 \end{cases} \qquad (2\text{-}6)$$

where E(θ) is given by equation (2-3) (with the subscript k suppressed). The integration in the elevation plane is restricted to the upper hemisphere because, for a lossy earth, there is no far-field radiation in the lower hemisphere. The directive gain $D(\theta) = 10 \log_{10} d(\theta)$ (dBi), computed by MITRE program MODIFIED IMAGES, is tabulated in tables 2 through 11 for the case h = 0.

The field $E_k(\theta)$, given by equation (2-3), may be rewritten as

$$E_k/A \sin\theta = (1 + |R_{k,v}| \cos\beta_k) + j\, |R_{k,v}| \sin\beta_\kappa \qquad (2\text{-}7)$$

where

β_κ = phase delay of the indirect array with respect to the direct ray

$$= \begin{cases} \beta_\kappa\,(\text{rad}) = \text{angle } R_{k,v}\,(\text{rad}) - (2\pi/\lambda_o)\, 2\, h \cos\theta \\ \beta_\kappa\,(\text{deg}) = \text{angle } R_{k,v}\,(\text{rad}) - (180/\pi)\,(2\pi/\lambda_o)\, 2\, h \cos\theta \end{cases} \qquad (2\text{-}8)$$

The argument α_k, of the complex quantity $E_k/A \sin\theta$, is given by

$$\alpha_k(\theta) = \text{arc tan}\,[\, |R_{k,v}| \sin\beta_\kappa / (1 + |R_{k,v}| \cos\beta_\kappa)] \qquad (2\text{-}9)$$

For homogeneous earth, $\alpha_k \equiv \alpha$ and $\beta_k = \beta$. The phase delay β and argument α are tabulated in tables 12, 13, and 14 for homogeneous very dry ground, medium dry ground,

Table 2. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Perfect Ground

CASE(1): Perfect Ground (Relative Dielectric Constant = 1.0, Conductivity = INFINITY S/m)

Grazing ang, deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	0.00000	4.7712	1.00000	0.00000	4.7712	1.00000	0.00000	4.7712
0.50	1.00000	0.00000	4.7709	1.00000	0.00000	4.7709	1.00000	0.00000	4.7709
1.00	1.00000	0.00000	4.7699	1.00000	0.00000	4.7699	1.00000	0.00000	4.7699
1.50	1.00000	0.00000	4.7682	1.00000	0.00000	4.7682	1.00000	0.00000	4.7682
2.00	1.00[illegible]	0.00000	4.7659	1.00000	0.00000	4.7659	1.00000	0.00000	4.7659
2.50	1.0[illegible]	0.00000	4.7629	1.00000	0.00000	4.7629	1.00000	0.00000	4.7629
3.00	1.0[illegible]	0.00000	4.7593	1.00000	0.00000	4.7593	1.00000	0.00000	4.7593
3.50	1.00000	0.00000	4.7550	1.00000	0.00000	4.7550	1.00000	0.00000	4.7550
4.00	1.00000	0.00000	4.7500	1.00000	0.00000	4.7500	1.00000	0.00000	4.7500
5.00	1.00000	0.00000	4.7381	1.00000	0.00000	4.7381	1.00000	0.00000	4.7381
6.00	1.00000	0.00000	4.7235	1.00000	0.00000	4.7235	1.00000	0.00000	4.7235
7.00	1.00000	0.00000	4.7062	1.00000	0.00000	4.7062	1.00000	0.00000	4.7062
8.00	1.00000	0.00000	4.6863	1.00000	0.00000	4.6863	1.00000	0.00000	4.6863
9.00	1.00000	0.00000	4.6636	1.00000	0.00000	4.6636	1.00000	0.00000	4.6636
10.00	1.00000	0.00000	4.6382	1.00000	0.00000	4.6382	1.00000	0.00000	4.6382
12.00	1.00000	0.00000	4.5793	1.00000	0.00000	4.5793	1.00000	0.00000	4.5793
14.00	1.00000	0.00000	4.5093	1.00000	0.00000	4.5093	1.00000	0.00000	4.5093
16.00	1.00000	0.00000	4.4280	1.00000	0.00000	4.4280	1.00000	0.00000	4.4280
18.00	1.00000	0.00000	4.3353	1.00000	0.00000	4.3353	1.00000	0.00000	4.3353
20.00	1.00000	0.00000	4.2309	1.00000	0.00000	4.2309	1.00000	0.00000	4.2309
22.00	1.00000	0.00000	4.1145	1.00000	0.00000	4.1145	1.00000	0.00000	4.1145
24.00	1.00000	0.00000	3.9858	1.00000	0.00000	3.9858	1.00000	0.00000	3.9858
26.00	1.00000	0.00000	3.8444	1.00000	0.00000	3.8444	1.00000	0.00000	3.8444
28.00	1.00000	0.00000	3.6899	1.00000	0.00000	3.6899	1.00000	0.00000	3.6899
30.00	1.00000	0.00000	3.5218	1.00000	0.00000	3.5218	1.00000	0.00000	3.5218
32.00	1.00000	0.00000	3.3396	1.00000	0.00000	3.3396	1.00000	0.00000	3.3396
34.00	1.00000	0.00000	3.1427	1.00000	0.00000	3.1427	1.00000	0.00000	3.1427
36.00	1.00000	0.00000	2.9304	1.00000	0.00000	2.9304	1.00000	0.00000	2.9304
38.00	1.00000	0.00000	2.7019	1.00000	0.00000	2.7019	1.00000	0.00000	2.7019
40.00	1.00000	0.00000	2.4563	1.00000	0.00000	2.4563	1.00000	0.00000	2.4563
42.00	1.00000	0.00000	2.1927	1.00000	0.00000	2.1927	1.00000	0.00000	2.1927
44.00	1.00000	0.00000	1.9099	1.00000	0.00000	1.9099	1.00000	0.00000	1.9099
46.00	1.00000	0.00000	1.6066	1.00000	0.00000	1.6066	1.00000	0.00000	1.6066
48.00	1.00000	0.00000	1.2814	1.00000	0.00000	1.2814	1.00000	0.00000	1.2814
50.00	1.00000	0.00000	0.9326	1.00000	0.00000	0.9326	1.00000	0.00000	0.9326
52.00	1.00000	0.00000	0.5581	1.00000	0.00000	0.5581	1.00000	0.00000	0.5581
54.00	1.00000	0.00000	0.1556	1.00000	0.00000	0.1556	1.00000	0.00000	0.1556
56.00	1.00000	0.00000	-0.2776	1.00000	0.00000	-0.2776	1.00000	0.00000	-0.2776
58.00	1.00000	0.00000	-0.7446	1.00000	0.00000	-0.7446	1.00000	0.00000	-0.7446
60.00	1.00000	0.00000	-1.2494	1.00000	0.00000	-1.2494	1.00000	0.00000	-1.2494
62.00	1.00000	0.00000	-1.7966	1.00000	0.00000	-1.7966	1.00000	0.00000	-1.7966
64.00	1.00000	0.00000	-2.3919	1.00000	0.00000	-2.3919	1.00000	0.00000	-2.3919
66.00	1.00000	0.00000	-3.0425	1.00000	0.00000	-3.0425	1.00000	0.00000	-3.0425
68.00	1.00000	0.00000	-3.7573	1.00000	0.00000	-3.7573	1.00000	0.00000	-3.7573
70.00	1.00000	0.00000	-4.5478	1.00000	0.00000	-4.5478	1.00000	0.00000	-4.5478
72.00	1.00000	0.00000	-5.4291	1.00000	0.00000	-5.4291	1.00000	0.00000	-5.4291
74.00	1.00000	0.00000	-6.4220	1.00000	0.00000	-6.4220	1.00000	0.00000	-6.4220
76.00	1.00000	0.00000	-7.5553	1.00000	0.00000	-7.5553	1.00000	0.00000	-7.5553
78.00	1.00000	0.00000	-8.8712	1.00000	0.00000	-8.8712	1.00000	0.00000	-8.8712
80.00	1.00000	0.00000	-10.4354	1.00000	0.00000	-10.4354	1.00000	0.00000	-10.4354
82.00	1.00000	0.00000	-12.3577	1.00000	0.00000	-12.3577	1.00000	0.00000	-12.3577
84.00	1.00000	0.00000	-14.8441	1.00000	0.00000	-14.8441	1.00000	0.00000	-14.8441
86.00	1.00000	0.00000	-18.3571	1.00000	0.00000	-18.3571	1.00000	0.00000	-18.3571
88.00	1.00000	0.00000	-24.3724	1.00000	0.00000	-24.3724	1.00000	0.00000	-24.3724
90.00	1.00000	0.00000	-142.4169	1.00000	0.00000	-142.4169	1.00000	0.00000	-142.4169

Table 3. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Sea Water

CASE(2): Sea Water (av salinity 20 deg C) (Relative Dielectric Constant = 70.0, Conductivity = 5.000000 S/m)

Grazing ang. deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		D.--ctive ga.
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.41413	-84.60905	0.0552	0.45255	-119.74591	-1.9779	0.53359	-139.11157	-3.8131
1.00	0.54337	-40.19803	2.3618	0.43633	-66.46782	1.1092	0.40790	-93.68260	-0.1391
1.50	0.65235	-25.98895	3.2328	0.52775	-42.49292	2.3677	0.44345	-62.20481	1.4593
2.00	0.72220	-19.24878	3.6853	0.60595	-31.06702	3.0450	0.50963	-45.17813	2.3508
2.50	0.76935	-15.30605	3.9605	0.66472	-24.51326	3.4653	0.56999	-35.33608	2.9163
3.00	0.80304	-12.71306	4.1441	0.70915	-20.26586	3.7498	0.61978	-29.02690	3.3049
3.50	0.82823	-10.87583	4.2744	0.74355	-17.28586	3.9541	0.66032	-24.64940	3.5872
4.00	0.84775	-9.50509	4.3708	0.77084	-15.07728	4.1070	0 69354	-21.43360	3.8004
5.00	0.87598	-7.59517	4.5013	0.81125	-12.01889	4.3178	0.74423	-17 02003	4.0983
6.00	0.89538	-6.32721	4.5819	0.83962	-9.99914	4.4527	0.78082	-14.12879	4.2929
7.00	0.90953	-5.42399	4.6327	0.86059	-8.56480	4.5426	0.80834	-12.08565	4.4261
8.00	0.92029	-4.74795	4.6638	0.87671	-7.49332	4.6032	0.82974	-10.56426	4.5196
9.00	0.92874	-4.22303	4.6809	0.88947	-6.66246	4.6433	0.84684	-9.38708	4.5854
10.00	0.93556	-3.80375	4.6873	0.89982	-5.99941	4.6681	0.86081	-8.44911	4.6310
12.00	0.94588	-3.17619	4.6759	0.91556	-5.00789	4.6844	0.88221	-7.04861	4.6785
14.00	0.95330	-2.72930	4.6399	0.92696	-4.30239	4.6681	0.89783	-6.05349	4.6846
16.00	0.95889	-2.39525	4.5840	0.93559	-3.77530	4.6271	0 90971	-5.31065	4.6603
18.00	0.96325	-2.13639	4.5110	0.94234	-3.36699	4.5656	0.91904	-4.73554	4.6118
20.00	0.96673	-1.93016	4.4222	0.94775	-3.04176	4.4860	0.92654	-4.27765	4.5426
22.00	0.96958	-1.76220	4.3186	0.95218	-2.77695	4.3899	0.93270	-3.90491	4.4550
24.00	0.97195	-1.62296	4.2005	0.95587	-2.55743	4.2779	0.93784	-3.59601	4.3501
26.00	0.97395	-1.50581	4.0680	0.95898	-2.37277	4.1507	0.94219	-3.33620	4.2288
28.00	0.97565	-1.40604	3.9211	0.96165	-2.21551	4.0082	0.94591	-3.11496	4.0913
30.00	0.97712	-1.32018	3.7596	0.96395	-2.08018	3.8505	0.94912	-2.92461	3.9380
32.00	0.97840	-1.24563	3.5830	0.96595	-1.96269	3.6773	0.95192	-2.75936	3.7686
34.00	0.97952	-1.18041	3.3911	0.96770	-1.85991	3.4882	0.95438	-2.61481	3.5828
36.00	0.98050	-1.12298	3.1831	0.96925	-1.76941	3.2828	0.95655	-2.48754	3.3803
38.00	0.98138	-1.07213	2.9585	0.97062	-1.68927	3.0604	0.95848	-2.37485	3.1605
40.00	0.98216	-1.02688	2.7163	0.97184	-1.61797	2.8203	0.96019	-2.27459	2.9227
42.00	0.98285	-0.98645	2.4558	0.97293	-1.55426	2.5616	0.96173	-2.18501	2.6661
44.00	0.98348	-0.95020	2.1758	0.97391	-1.49713	2.2832	0.96311	-2.10468	2.3895
46.00	0.98404	-0.91760	1.8750	0.97480	-1.44576	1.9839	0.96435	-2.03245	2.0918
48.00	0.98455	-0.88820	1.5520	0.97559	-1.39944	1.6622	0.96547	-1.96733	1.7717
50.00	0.98501	-0.86165	1.2052	0.97632	-1.35760	1.3166	0.96649	-1.90851	1.4274
52.00	0.98542	-0.83763	0.8325	0.97697	-1.31975	0.9450	0.96741	-1.85530	1.0570
54.00	0.98580	-0.81588	0.4317	0.97756	-1.28548	0.5451	0.96824	-1.80712	0.6582
56.00	0.98614	-0.79618	0.0001	0.97809	-1.25444	0.1144	0.96899	-1.76347	0.2285
58.00	0.98645	-0.77833	-0.4656	0.97858	-1.22632	-0.3505	0.96968	-1.72394	-0.2355
60.00	0.98672	-0.76217	-0.9692	0.97902	-1.20086	-0.8534	0.97030	-1.68815	-0.7375
62.00	0.98698	-0.74756	-1.5153	0.97942	-1.17784	-1.3988	0.97086	-1.65579	-1.2822
64.00	0.98721	-0.73438	-2.1096	0.97978	-1.15708	-1.9926	0.97136	-1.62660	-1.8753
66.00	0.98741	-0.72253	-2.7593	0.98010	-1.13839	-2.6417	0.97182	-1.60033	-2.5238
68.00	0.98759	-0.71190	-3.4732	0.98039	-1.12165	-3.3552	0.97223	-1.57679	-3.2368
70.00	0.98776	-0.70242	-4.2630	0.98065	-1.10672	-4.1445	0.97259	-1.55580	-4.0256
72.00	0.98790	-0.69403	-5.1437	0.98088	-1.09349	-5.0249	0.97292	-1.53722	-4.9056
74.00	0.98803	-0.68666	-6.1361	0.98108	-1.08189	-6.0169	0.97320	-1.52090	-5.8972
76.00	0.98814	-0.68027	-7.2688	0.98125	-1.07181	-7.1494	0.97345	-1.50674	-7.0293
78.00	0.98824	-0.67481	-8.5844	0.98140	-1.06321	-8.4646	0.97366	-1.49464	-8.3443
80.00	0.98832	-0.67024	-10.1482	0.98153	-1.05602	-10.0283	0.97383	-1.48454	-9.9077
82.00	0.98838	-0.66655	-12.0702	0.98163	-1.05020	-11.9501	0.97398	-1.47635	-11.8294
84.00	0.98843	-0.66370	-14.5564	0.98171	-1.04571	-14.4362	0.97408	-1.47004	-14.3153
86.00	0.98847	-0.66167	-18.0692	0.98176	-1.04252	-17.9489	0.97416	-1.46555	-17.8280
88.00	0.98849	-0.66046	-24.0845	0.98179	-1.04061	-23.9641	0.97421	-1.46288	-23.8431
90.00	0.98849	-0.66006	-142.1289	0.98181	-1.03998	-142.0085	0.97423	-1.46199	-141.8875

Table 4. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Fresh Water

CASE(3): Fresh Water (Relative Dielectric Constant = 80.0, Conductivity = 0.030000 S/m)

Grazing ang, deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.83894	-175.49135	-13.8869	0.85125	-178.03345	-14.8065	0.85354	-179.00006	-14.9947
1.00	0.70309	-170.78516	-8.5548	0.72332	-175.99016	-9.4131	0.72713	-177.96196	-9.5900
1.50	0.58817	-165.66431	-5.6783	0.61235	-173.78383	-6.4785	0.61698	-176.84261	-6.6446
2.00	0.49104	-159.87224	-3.7860	0.51541	-171.30666	-4.5318	0.52021	-175.58774	-4.6876
2.50	0.40955	-153.09621	-2.4194	0.43031	-168.41162	-3.1146	0.43461	-174.12224	-3.2607
3.00	0.34241	-144.96724	-1.3761	0.35541	-164.88354	-2.0242	0.35845	-172.33318	-2.1612
3.50	0.28915	-135.11427	-0.5493	0.28957	-160.38875	-1.1536	0.29041	-170.03693	-1.2822
4.00	0.24989	-123.33838	0.1240	0.23217	-154.38623	-0.4398	0.22951	-166.90907	-0.5605
5.00	0.21363	-95.87459	1.1563	0.14395	-133.80182	0.6650	0.12723	-154.89259	0.5588
6.00	0.22377	-70.96504	1.9109	0.10523	-94.41348	1.4823	0.05995	-114.20327	1.3887
7.00	0.25760	-53.81433	2.4851	0.12537	-56.40618	2.1111	0.07561	-46.19757	2.0287
8.00	0.29849	-42.81393	2.9346	0.17040	-37.19942	2.6085	0.13010	-24.50036	2.5360
9.00	0.33926	-35.51364	3.2939	0.21803	-27.61649	3.0100	0.18355	-16.79543	2.9463
10.00	0.37745	-30.39718	3.5855	0.26268	-22.09290	3.3389	0.23182	-12.94742	3.2831
12.00	0.44412	-23.74165	4.0219	0.34006	-16.02251	3.8385	0.31357	-9.08301	3.7963
14.00	0.49885	-19.59953	4.3212	0.40325	-12.73470	4.1892	0.37949	-7.11487	4.1580
16.00	0.54395	-16.76267	4.5256	0.45531	-10.65257	4.4360	0.43352	-5.90497	4.4141
18.00	0.58152	-14.69151	4.6603	0.49875	-9.20506	4.6061	0.47852	-5.07824	4.5922
20.00	0.61318	-13.10976	4.7409	0.53544	-8.13541	4.7168	0.51651	-4.47403	4.7096
22.00	0.64014	-11.86098	4.7778	0.56679	-7.31043	4.7795	0.54895	-4.01156	4.7781
24.00	0.66333	-10.84973	4.7780	0.59383	-6.65377	4.8021	0.57693	-3.64546	4.8057
26.00	0.68345	-10.01432	4.7463	0.61736	-6.11834	4.7899	0.60128	-3.34817	4.7980
28.00	0.70103	-9.31297	4.6863	0.63797	-5.67340	4.7470	0.62263	-3.10191	4.7590
30.00	0.71650	-8.71638	4.6002	0.65616	-5.29799	4.6761	0.64147	-2.89465	4.6916
32.00	0.73019	-8.20329	4.4897	0.67229	-4.97726	4.5791	0.65818	-2.71794	4.5977
34.00	0.742[illegible]	-7.75797	4.3560	0.68666	-4.70039	4.4573	0.67308	-2.56564	4.4787
36.00	0.753[illegible]	-7.36843	4.1996	0.69954	-4.45931	4.3117	0.68643	-2.43321	4.3356
38.00	0.763[illegible]2	-7.02543	4.0209	0.71111	-4.24784	4.1427	0.69843	-2.31718	4.1688
40.00	0.771[illegible]	-6.72169	3.8199	0.72153	-4.06120	3.9504	0.70925	-2.21488	3.9785
42.00	0.77974	-6.45144	3.5962	[illegible]	-3.6[illegible]	3.7347	0.71903	-2.12418	3.7646
44.00	[illegible]592	-6.20999	3.3494	0.7395[illegible]	-3.74801	3.4950	0.72790	-2.04342	3.5266
46.00	[illegible]43	-5.99353	3.0786	0.74726	-3.61599	3.2307	0.73595	-1.97122	3.2639
48.00	[illegible]5	-5.79893	2.7827	0.75432	-3.49753	2.9407	0.74327	-1.90647	2.9753
50.00	[illegible]473	-5.62360	2.4604	0.76074	-3.39097	2.6238	0.74994	-1.84825	2.6597
52.00	0.[illegible]32	-5.46533	2.1100	0.76658	-3.29494	2.2783	0.75601	-1.79581	2.3153
54.00	0.8[illegible]98	-5.32233	1.7294	0.77191	-3.20828	1.9022	0.76155	-1.74851	1.9402
56.00	0.[illegible]13	-5.19300	1.3161	0.77677	-3.13001	1.4930	0.76660	-1.70580	1.5320
58.00	0.[illegible]83	-5.07605	0.8671	0.78119	-3.05930	1.0477	0.77119	-1.66723	1.0875
60.00	0.8[illegible]18	-4.97033	0.3786	0.78521	-2.99544	0.5626	0.77537	-1.63241	0.6033
62.00	0.8[illegible]323	-4.87488	-0.1538	0.78886	-2.93784	0.0332	0.77916	-1.60100	0.0746
64.00	0.83098	-4.78886	-0.7358	0.79217	-2.88597	-0.5460	0.78260	-1.57273	-0.5039
66.00	0.83347	-4.71154	-1.3744	0.79516	-2.83939	-1.1820	0.78570	-1.54735	-1.1394
68.00	0.83571	-4.64232	-2.0783	0.79784	-2.79770	-1.8837	0.78849	-1.52464	-1.8405
70.00	0.83771	-4.58065	-2.8592	0.80024	-2.76058	-2.6625	0.79099	-1.50442	-2.6189
72.00	0.83948	-4.52607	-3.7320	0.80237	-2.72776	-3.5336	0.79321	-1.48654	-3.4895
74.00	0.84104	-4.47819	-4.7174	0.80424	-2.69897	-4.5174	0.79515	-1.47086	-4.4730
76.00	0.84240	-4.43667	-5.8441	0.80587	-2.67401	-5.6428	0.79685	-1.45728	-5.5980
78.00	0.84356	-4.40123	-7.1545	0.80727	-2.65272	-6.9519	0.79830	-1.44568	-6.9069
80.00	0.84453	-4.37162	-8.7140	0.80843	-2.63494	-8.5105	0.79951	-1.43600	-8.4652
82.00	0.84531	-4.34765	-10.6325	0.80938	-2.62054	-10.4282	0.80050	-1.42816	-10.3828
84.00	0.84592	-4.32916	-13.1160	0.81011	-2.60945	-12.9111	0.80126	-1.42212	-12.8655
86.00	0.84635	-4.31604	-16.6270	0.81063	-2.60157	-16.4216	0.80179	-1.41783	-16.3759
88.00	0.84661	-4.30820	-22.6410	0.81094	-2.59687	-22.4354	0.80212	-1.41527	-22.3897
90.00	0.84670	-4.30559	-140.6851	0.81104	-2.59530	-140.4794	0.80222	-1.41442	-140.4336

Table 5. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Wet Ground

CASE(4): Wet Ground (Relative Dielectric Constant = 30.0, Conductivity = 0.010000 S/m)

Grazing ang, deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.89913	-177.53148	-17.0270	0.90572	-178.94031	-17.7075	0.90688	-179.46329	-17.8355
1.00	0.80820	-175.02328	-11.4400	0.81997	-177.86513	-12.0893	0.82206	-178.91888	-12.2117
1.50	0.72605	-172.43419	-8.3347	0.74170	-176.75836	-8.9534	0.74449	-178.35872	-9.0705
2.00	0.65170	-169.71970	-6.2364	0.67002	-175.60228	-6.8256	0.67331	-177.77399	-6.9375
2.50	0.58435	-166.83031	-4.6839	0.60417	-174.37682	-5.2445	0.60777	-177.15462	-5.3513
3.00	0.52334	-163.70963	-3.4721	0.54353	-173.05827	-4.0053	0.54724	-176.48869	-4.1072
3.50	0.46815	-160.29240	-2.4921	0.48756	-171.61778	-2.9990	0.49118	-175.76157	-3.0962
4.00	0.41835	-156.50290	-1.6792	0.43581	-170.01923	-2.1609	0.43915	-174.95476	-2.2536
5.00	0.33389	-147.44798	-0.4018	0.34352	-166.14746	-0.8364	0.34562	-172.99739	-0.9206
6.00	0.26892	-135.76707	0.5602	0.26445	-160.85295	0.1686	0.26413	-170.29680	0.0924
7.00	0.22391	-120.91679	1.3121	0.19754	-153.01488	0.9597	0.19295	-166.18292	0.8907
8.00	0.19972	-103.51066	1.9153	0.14340	-140.41916	1.5987	0.13122	-158.98991	1.5364
9.00	0.19508	-85.89762	2.4085	0.10591	-119.41214	2.1248	0.08013	-143.57930	2.0686
10.00	0.20536	-70.66362	2.8178	0.09252	-89.66296	2.5641	0.04933	-104.43576	2.5136
[illegible]	0.24836	-49.87074	3.4513	0.12927	-45.24990	3.2506	0.09106	-31.45258	3.2102
[illegible]	[illegible]	-38.06417	3.9087	0.18885	-28.48850	3.7532	0.15944	-17.03982	3.7215
[illegible]	[illegible]	[illegible]	4.2425	0.24531	-20.98231	4.1257	0.22022	-11.96165	4.1015
[illegible]	[illegible]	[illegible]	4.4847	0.29534	-16.80100	4.4014	0.27283	-9.37218	4.3839
[illegible]	[illegible]	[illegible]	4.6560	0.33920	-14.13327	4.6020	0.31845	-7.79073	4.5902
[illegible]	[illegible]	[illegible]	4.7703	0.37768	-12.27667	4.7419	0.35825	-6.71791	4.7352
[illegible]	[illegible]	[illegible]	4.8372	0.41157	-10.90581	4.8315	0.39320	-5.93881	4.8293
[illegible]	[illegible]	[illegible]	4.8636	0.44156	-9.84976	4.8779	0.42406	-5.34553	4.8798
[illegible]	[illegible]	[illegible]	4.8543	0.46822	-9.01013	4.8866	0.45147	-4.87779	4.8920
[illegible]	[illegible]	[illegible]	4.8130	0.49203	-8.32616	4.8613	0.47592	-4.49921	4.8699
[illegible]	[illegible]	[illegible]	4.7422	0.51338	-7.75822	4.8049	0.49784	-4.18642	4.8165
[illegible]	[illegible]	-12.54901	4.6438	0.53259	-7.27932	4.7195	0.51755	-3.92373	4.7337
[illegible]	[illegible]	-11.87998	4.5191	0.54993	-6.87042	4.6065	0.53535	-3.70017	4.6231
[illegible]	0.62922	-11.29714	4.3688	0.56563	-6.51766	4.4668	0.55145	-3.50782	4.4856
[illegible]	0.64179	-10.78574	4.1934	0.57987	-6.21069	4.3010	0.56607	-3.34063	4.3218
42.00	0.65321	-10.33432	3.9929	0.59283	-5.94164	4.1093	0.57935	-3.19476	4.1318
44.00	0.66361	-9.93383	3.7670	0.60462	-5.70440	3.8914	0.59146	-3.06617	3.9156
46.00	0.67308	-9.57699	3.5152	0.61539	-5.49416	3.6469	0.60250	-2.95239	3.6726
48.00	0.68173	-9.25793	3.2367	0.62522	-5.30707	3.3750	0.61258	-2.85126	3.4020
50.00	0.68963	-8.97183	2.9301	0.63420	-5.14000	3.0745	0.62179	-2.76106	3.1028
52.00	0.69684	-8.71472	2.5940	0.64241	-4.99043	2.7440	0.63022	-2.65038	2.7734
54.00	0.70343	-8.48326	2.2264	0.64991	-4.85622	2.3815	0.63791	-2.60806	2.4120
56.00	0.70945	-8.27470	1.8250	0.65677	-4.73565	1.9847	0.64495	-2.54314	2.0161
58.00	0.71495	-8.08668	1.3868	0.66303	-4.62724	1.5508	0.65137	-2.48481	1.5830
60.00	0.71995	-7.91719	0.9082	0.66873	-4.52977	1.0760	0.65722	-2.43239	1.1090
62.00	0.72450	-7.76456	0.3847	0.67393	-4.44217	0.5560	0.66255	-2.38532	0.5698
64.00	0.72863	-7.62733	-0.1892	0.67864	-4.36356	-0.0147	0.66738	-2.34310	0.0197
66.00	0.73237	-7.50424	-0.8204	0.68290	-4.29319	-0.6431	0.67175	-2.30531	-0.6081
68.00	0.73573	-7.39425	-1.5179	0.68673	-4.23040	-1.3379	0.67569	-2.27161	-1.3024
70.00	0.73874	-7.29642	-2.2928	0.69017	-4.17462	-2.1106	0.67921	-2.24170	-2.0746
72.00	0.74141	-7.20997	-3.1605	0.69322	-4.12541	-2.9762	0.68234	-2.21531	-2.9397
74.00	0.74377	-7.13423	-4.1413	0.69591	-4.08234	-3.9552	0.68510	-2.19221	-3.9184
76.00	0.74582	-7.06863	-5.2640	0.69825	-4.04506	-5.0763	0.68750	-2.17224	-5.0392
78.00	0.74757	-7.01268	-6.5710	0.70025	-4.01331	-6.3820	0.68955	-2.15522	-6.3446
80.00	0.74904	-6.96598	-8.1277	0.70193	-3.98681	-7.9375	0.69127	-2.14103	-7.8999
82.00	0.75023	-6.92821	-10.0439	0.70329	-3.96540	-9.8528	0.69267	-2.12955	-9.8150
84.00	[illegible]	-6.89909	-12.5256	0.70434	-3.94890	-12.3338	0.69375	-2.12072	-12.2959
86.00	0.75180	-6.87843	-16.0353	0.70509	-3.93720	-15.8430	0.69451	-2.11445	-15.8049
88.00	0.75219	-6.86609	-22.0486	0.70553	-3.93021	-21.8560	0.69497	-2.11071	-21.8179
90.00	0.75232	-6.86198	-140.0924	0.70568	-3.92788	-139.8997	0.69513	-2.10946	-139.8616

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Table 6. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Medium Dry Ground

CASE(5): Medium Dry Ground (Relative Dielectric Constant = 15.0, Conductivity = 0.001000 S/m)

Grazing ang, deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.93213	-179.62875	-19.6298	0.93235	-179.85101	-19.6599	0.93239	-179.92548	-19.6642
1.00	0.86872	-179.25478	-13.9006	0.86914	-179.70094	-13.9296	0.86920	-179.85040	-13.9338
1.50	0.80935	-178.87526	-10.6617	0.80994	-179.54865	-10.6897	0.81003	-179.77422	-10.6937
2.00	0.75367	-178.48729	-8.[illegible]	0.75440	-179.39296	-8.4649	0.75450	-179.69635	-8.4688
2.50	0.70135	-178.08771	-6.7675	0.70218	-179.23765	-6.7934	0.70230	-179.61615	-6.[illegible]
3.00	0.65210	-177.67305	-5.4447	0.65300	-179.[illegible]	-5.4696	0.65314	-179.53296	-5.4732
3.50	0.60566	-177.23953	-4.3601	0.606[illegible]	-178.89244	-4.3841	0.60676	-179.44598	-4.3875
4.00	0.56181	-176.78276	-3.4485	0.56280	-178.70926	-3.4715	0.56294	-179.35435	-3.4749
5.00	0.48110	-175.77838	-1.9899	0.48206	-178.30653	-2.0111	0.48220	-179.15292	-2.0142
6.00	0.40858	-174.60728	-0.8655	0.40940	-177.83694	-0.8851	0.40952	-178.91803	-0.8879
7.00	0.34315	-173.19183	0.0324	0.34370	-177.26906	0.0144	0.34378	-178.63399	0.0118
8.00	0.28393	-171.41025	0.7674	0.28405	-176.55312	0.7510	0.28406	-178.27177	0.7486
9.00	0.23021	-169.05791	1.3804	0.22967	-175.60406	1.3653	0.22960	-177.80066	1.3632
10.00	0.18152	-165.76236	1.8985	0.17997	-174.26274	1.8848	0.17974	-177.12831	1.8829
12.00	0.09880	-152.40816	2.7227	0.09278	-168.52808	2.7114	0.09188	-[illegible]74.22993	2.7098
14.00	0.04773	-104.73233	3.3411	[illegible]2434	-130.07385	3.3320	0.01867	-[illegible]50.04689	3.3307
16.00	0.0[illegible]016	-41.07962	3.8124	[illegible]5264	-20.70421	3.8052	0.04959	-10.82862	3.[illegible]042
18.00	0.[illegible]1862	-22.68665	4.1732	0.10732	-9.91372	4.1678	0.10558	-5.02667	4.1670
20.00	0.16531	-15.87302	4.4476	0.15629	-6.71162	4.4437	0.15494	-3.38414	4.4431
22.00	0.[illegible]756	-12.39734	4.6522	0.19972	-5.17379	4.6496	0.19856	-2.60329	4.6492
24.00	0.24545	-10.29028	4.7986	0.23834	-4.26571	4.7972	0.23729	-2.[illegible]414	4.7970
26.00	0.27944	-8.87327	4.8954	0.27282	-3.66376	4.8951	0.27185	-1.64045	4.8951
28.00	0.30998	-7.85288	4.9489	0.30374	-3.23416	4.9496	0.30282	-1.62402	4.9497
30.00	0.3375[illegible]	-7.08192	4.9637	0.33155	-2.91154	4.9652	0.33068	-1.46163	4.9655
32.00	0.36240	-6.47844	4.9433	0.35667	-2.66011	4.9456	0.35583	-1.33516	4.9460
34.00	0.38495	-5.99314	4.8902	0.3794[illegible]	-2.45859	4.8933	0.37860	-1.23384	4.8938
36.00	0.40543	-5.59457	4.8065	0.40006	-2.29350	4.8102	0.39927	-1.15087	4.8108
38.00	0.42408	-5.26169	4.6933	0.41885	-2.15591	4.6976	0.41808	-1.08174	4.6983
40.00	0.44108	-4.97986	4.5515	0.43598	-2.03961	4.5564	0.43523	-1.02333	4.5571
42.00	0.45662	-4.73658	4.3816	0.45162	-1.94018	4.3870	0.45088	-0.97340	4.3878
44.00	0.47082	-4.53010	4.1836	0.46592	-1.85437	4.1894	0.46520	-0.93031	4.1903
46.00	0.48383	-4.34859	3.9572	0.47901	-1.77974	3.9635	0.47830	-0.89284	3.9644
48.00	0.49574	-4.18956	3.7018	0.49100	-1.71440	3.7085	0.49031	-0.86005	3.7094
50.00	0.50666	-4.04950	3.4165	0.50199	-1.65691	3.4234	0.50130	-0.83119	3.4244
52.00	0.51667	-3.92562	3.0997	0.51206	-1.60609	3.1070	0.51138	-0.80569	3.1080
54.00	0.52584	-3.81569	2.7498	0.52128	-1.56102	2.7574	0.52061	-0.78308	2.7585
56.00	0.53423	-3.71790	2.3645	0.52972	-1.52095	2.3723	0.52906	-0.76297	2.3735
58.00	0.54191	-3.63075	1.9410	0.53744	-1.48525	1.9491	0.53679	-0.74506	1.9502
60.00	0.54892	-3.55300	1.4758	0.54449	-1.45343	1.4840	0.54385	-0.72909	1.4852
62.00	0.55531	-3.48364	0.9645	0.55092	-1.42504	0.9729	0.55028	-0.71485	0.9741
64.00	0.56111	-3.42181	0.4016	0.55676	-1.39974	0.4102	0.556[illegible]2	-0.70216	0.4115
66.00	0.56636	-3.36676	-0.2197	0.56204	-1.37723	-0.2109	[illegible]	-0.69087	-0.2097
68.00	0.57110	-3.31791	-0.9081	0.56680	-1.35726	-0.8992	[illegible]	-0.68085	-0.897[illegible]
70.00	0.57534	-3.27473	-1.6751	0.57107	-1.33960	-1.6661	0.57044	-0.67199	[illegible]
72.00	0.57912	-3.23677	-2.5356	0.57487	-1.32409	-2.5265	0.57424	-0.66421	-2.52[illegible]
74.00	0.58245	-3.20368	-3.5102	0.57821	-1.31057	-3.5009	0.57759	-0.65743	-3.4996
76.00	0.58535	-3.17514	-4.6275	0.58112	-1.29891	-4.6181	0.58051	-0.65158	-4.6168
78.00	0.58783	-3.15089	-5.9290	0.58362	-1.28900	-5.9204	0.58300	-0.64662	-5.9[illegible]
80.00	0.58991	-3.13071	-7.4826	0.58571	-1.28076	-7.4731	0.58509	-0.64[illegible]	-7.4717
82.00	0.59159	-3.11442	-9.3956	0.58741	-1.27411	-9.3861	0.58679	-0.639[illegible]	-9.3847
84.00	0.59290	-3.10190	-11.87[illegible]	0.58872	-1.26900	-11.8653	0.58810	-0.636[illegible]8	-11.8639
86.00	0.59382	-3.09303	-15.38[illegible]	0.58965	-1.26537	-15.3732	0.58904	-0.63477	-15.3718
88.00	0.59438	-3.08773	-21.39[illegible]	0.59021	-1.26321	-21.3855	0.58959	-0.63368	-21.3841
90.00	0.59456	-3.08597	-139.43[illegible]	0.59039	-1.26249	-139.4290	0.58978	-0.63332	-139.4276

Table 7. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Very Dry Ground

CASE(6): Very Dry Ground (Relative Dielectric Constant = 3.0, Conductivity = 0.000100 S/m)

Grazing ang. deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.96368	-179.94652	-22.4733	0.96366	-179.97874	-22.4658	0.96365	-179.98940	-22.4647
1.00	0.92867	-179.89288	-16.6108	0.92861	-179.95746	-16.6034	0.92861	-179.97874	-16.6023
1.50	0.89489	-179.83899	-13.2454	0.89482	-179.93604	-13.2381	0.89480	-179.96805	-13.2371
2.00	0.86229	-179.78468	-10.9016	0.86220	-179.91446	-10.8944	0.86218	-179.95728	-10.8934
2.50	0.83083	-179.72981	-9.1168	0.83071	-179.89267	-9.1098	0.83069	-179.94640	-9.1088
3.00	0.80043	-179.67427	-7.6853	0.80030	-179.87059	-7.6784	0.80028	-179.93536	-7.6774
3.50	0.77106	-179.61789	-6.4970	0.77091	-179.84819	-6.4903	0.77089	-179.92416	-6.4893
4.00	0.74267	-179.56052	-5.4866	0.74250	-179.82538	-5.4800	0.74248	-179.91277	-5.4790
5.00	0.68866	-179.44214	-3.8436	0.68846	-179.77832	-3.8372	0.68844	-179.88928	-3.8362
6.00	0.63808	-179.31784	-2.5506	0.63786	-179.72888	-2.5445	0.63783	-179.86458	-2.5436
7.00	0.59066	-179.18614	-1.4982	0.59042	-179.67650	-1.4923	0.59038	-179.83841	-1.4915
8.00	0.54613	-179.04533	-0.6212	0.54587	-179.62045	-0.6155	0.54583	-179.81039	-0.6147
9.00	0.50427	-178.89354	0.1225	0.50400	-179.56001	0.1279	0.50396	-179.78020	0.1287
10.00	0.46488	-178.72852	0.7614	0.46460	-179.49428	0.7666	0.46455	-179.74736	0.7673
12.00	0.39279	-178.34776	1.8007	0.39247	-179.34251	[illegible]	0.39243	-179.67151	1.8060
14.00	0.32856	-177.87482	2.6045	0.32822	-179.15382	2.6086	0.32817	-179.57721	2.6092
16.00	0.27116	-177.26640	3.2364	0.27079	-178.91084	3.2400	0.27073	-179.45573	3.2406
18.00	0.21971	-176.45151	3.7371	0.21929	-178.58498	3.7402	0.21923	-179.29282	3.7407
20.00	0.17350	-175.30396	4.1339	0.17298	-178.12523	4.1365	0.17291	-179.06290	4.1369
22.00	0.13192	-173.57455	4.4460	0.13124	-177.43015	[illegible]	0.13115	-178.71512	4.4484
24.00	0.09453	-170.69418	4.6872	0.09356	-176.26411	[illegible]	0.09342	-178.13107	4.6891
26.00	0.06111	-165.03671	4.8678	0.05950	-173.92628	[illegible]	0.05927	-176.95660	4.8691
28.00	0.03234	-149.84064	4.9955	0.02889	-167.02365	4.9962	0.02836	-173.43538	4.9963
30.00	0.01670	-89.02242	5.0762	0.00667	-89.61656	5.0765	0.00333	-89.80935	5.0766
32.00	0.03114	-33.37439	5.1145	0.02660	-14.90757	5.1144	0.02589	-7.59295	5.1144
34.00	0.05247	-19.53377	5.1139	0.04960	-8.12801	5.1133	0.04917	-4.08820	5.1133
36.00	0.07309	-14.21117	5.0769	0.07081	-5.81511	5.0760	0.07048	-2.91726	5.0758
38.00	0.09224	-11.45934	5.0055	0.09024	-4.65907	5.0043	0.08996	-2.33506	5.0041
40.00	0.10988	-9.79510	4.9013	0.10804	-3.97016	4.8997	0.10777	-1.98888	4.8994
42.00	0.12609	-8.68840	4.7650	0.12433	-3.51560	4.7630	0.12407	-1.76077	4.7628
44.00	0.14097	-7.90459	4.5971	0.13924	-3.19538	4.5949	0.13900	-1.60012	4.5946
46.00	0.15461	-7.32409	4.3979	0.15291	-2.95895	4.3953	0.15266	-1.48160	4.3950
48.00	0.16713	-6.87971	4.1668	[illegible]	-2.77839	4.1640	0.16519	-1.39111	4.1636
50.00	0.17860	-6.53081	3.9033	0.17690	-2.63688	3.9003	0.17665	-1.32022	3.8998
52.00	0.18911	-6.25140	3.6063	0.18740	-2.52371	3.6030	0.18715	-1.26353	3.6025
54.00	0.19873	-6.02407	3.2741	0.19701	-2.43174	3.2706	0.19676	-1.21747	3.2701
56.00	0.20753	-5.83675	2.9047	0.20579	-2.35602	2.9010	0.20554	-1.17956	2.9004
58.00	0.21557	-5.68081	2.4955	0.21382	-2.29304	2.4916	0.21356	-1.14802	2.4910
60.00	0.22290	-5.54992	2.0431	0.22113	-2.24022	2.0390	0.22087	-1.12158	2.0384
62.00	0.27957	-5.43935	1.5432	0.22778	-2.19562	1.5390	0.22752	-1.09925	1.5383
64.00	0.23562	-5.34550	0.9906	0.23381	-2.15778	0.9862	0.23355	-1.08031	0.9856
66.00	0.24109	-5.26556	0.3785	0.23927	-2.12557	0.3740	0.23900	-1.06418	0.3733
68.00	0.24601	-5.19737	-0.3018	0.24417	-2.09810	-0.3065	0.24390	-1.05043	-0.3072
70.00	0.25041	-5.13919	-1.0616	0.24856	-2.07467	-1.0664	0.24829	-1.03871	-1.0671
72.00	0.25432	-5.08965	-1.9158	0.25245	-2.05472	-1.9207	0.25219	-1.02873	-1.9214
74.00	0.25776	-5.04764	-2.8848	0.25588	-2.03782	-2.8898	0.25561	-1.02027	-2.8905
76.00	0.26076	-5.01229	-3.9974	0.25886	-2.02359	-4.0025	0.25859	-1.01315	-4.0032
78.00	0.26332	-4.98288	-5.2957	0.26141	-2.01176	-5.3008	0.26114	-1.00723	-5.3016
80.00	0.26546	-4.95885	-6.8451	0.26355	-2.00209	-6.8503	0.26327	-1.00239	-6.8510
82.00	0.26720	-4.93974	-8.7555	0.26528	-1.99440	-8.7607	0.26500	-0.99854	-8.7615
84.00	0.26854	-4.92521	-11.2327	0.26662	-1.98856	-11.2380	0.26634	-0.99562	-11.2387
86.00	0.26949	-4.91501	-14.7391	0.26756	-1.98446	-14.7444	0.26729	-0.99357	-14.7452
88.00	0.27006	-4.90896	-20.7505	0.26813	-1.98203	-20.7559	0.26785	-0.99235	-20.7566
90.00	0.27025	-4.90695	-138.7937	0.26832	-1.98122	-138.7991	0.26804	-0.99195	-138.7999

Table 8. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Pure Water, 20°C

CASE(7): Pure water, 20 deg C (Relative Dielectric Constant = 80.0, Conductivity is defined below)

Grazing ang. deg	FREQUENCY = 6.0 MHz. COND = 0.000037 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. COND = 0.000500 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. COND = 0.001700 S/m Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.85435	-179.99883	-15.0628	0.85435	-179.96648	-15.0627	0.85435	-179.94301	-15.0626
1.00	0.72848	-179.99762	-9.6541	0.72848	-179.93167	-9.6540	0.72848	-179.88383	-9.6539
1.50	0.61863	-179.99631	-6.7048	0.61863	-179.89417	-6.7047	0.61863	-179.82008	-6.7046
2.00	0.52193	-179.99483	-4.7442	0.52193	-179.85214	-4.7442	0.52193	-179.74864	-4.7440
2.50	0.43616	-179.99310	-3.3139	0.43616	-179.80309	-3.3138	0.43616	-179.66524	-3.3137
3.00	0.35957	-179.99101	-2.2112	0.35957	-179.74315	-2.2111	0.35957	-179.56335	-2.2110
3.50	0.29077	-179.98831	-1.3291	0.29077	-179.66603	-1.3291	0.29077	-179.43227	-1.3290
4.00	0.22862	-179.98460	-0.6045	0.22862	-179.56027	-0.6045	0.22863	-179.25249	-0.6044
5.00	0.12081	-179.96973	0.5199	0.12082	-179.13477	0.5199	0.12083	-178.52922	0.5200
6.00	0.03052	-179.87846	1.3544	0.03057	-176.53107	1.3544	0.03066	-174.11409	1.3545
7.00	0.04617	-0.08025	1.9984	0.04621	-2.29138	1.9984	0.04630	-3.89019	1.9985
8.00	0.11210	-0.03271	2.5092	0.11212	-0.93440	2.5093	0.11216	-1.58797	2.5093
9.00	0.16937	-0.02130	2.9227	0.16938	-0.60844	2.9228	0.16942	-1.03415	2.9228
10.00	0.21956	-0.01610	3.2624	0.21957	-0.45996	3.2624	0.21960	-0.78182	3.2625
12.00	0.30334	-0.01112	3.7806	0.30335	-0.31766	3.7806	0.30337	-0.53996	3.7806
14.00	0.37041	-0.00865	4.1464	0.37042	-0.24725	4.1464	0.37044	-0.42026	4.1464
16.00	0.42524	-0.00716	4.4059	0.42525	-0.20452	4.4059	0.42527	-0.34765	4.4060
18.00	0.47086	-0.00614	4.5868	0.47087	-0.17554	4.5869	0.47088	-0.29839	4.5869
20.00	0.50934	-0.00541	4.7068	0.50935	-0.15445	4.7068	0.50936	-0.26255	4.7068
22.00	0.54220	-0.00484	4.7775	0.54221	-0.13836	4.7775	0.54222	-0.23520	4.7775
24.00	0.57055	-0.00440	4.8069	0.57055	-0.12565	4.8069	0.57057	-0.21360	4.8069
26.00	0.59521	-0.00404	4.8009	0.59522	-0.11535	4.8009	0.59523	-0.19609	4.8009
28.00	0.61684	-0.00374	4.7633	0.61684	-0.10683	4.7633	0.61685	-0.18160	4.7633
30.00	0.63592	-0.00349	4.6972	0.63592	-0.09966	4.6971	0.63594	-0.16942	4.6971
32.00	0.65285	-0.00327	4.6044	0.65286	-0.09356	4.6044	0.65287	-0.15904	4.6044
34.00	0.66795	-0.00309	4.4865	0.66796	-0.08830	4.4865	0.66797	-0.15010	4.4864
36.00	0.68148	-0.00293	4.3443	0.68148	-0.08373	4.3443	0.68149	-0.14234	4.3443
38.00	0.69363	-0.00279	4.1784	0.69364	-0.07973	4.1783	0.69365	-0.13553	4.1783
40.00	0.70460	-0.00267	3.9888	0.70460	-0.07620	3.9888	0.70461	-0.12954	3.9888
42.00	0.71452	-0.00256	3.7756	0.71452	-0.07308	3.7756	0.71453	-0.12422	3.7756
44.00	0.72350	-0.00246	3.5383	0.72351	-0.07030	3.5382	0.72352	-0.11949	3.5382
46.00	0.73167	-0.00237	3.2760	0.73167	-0.06781	3.2760	0.73168	-0.11527	3.2760
48.00	0.73909	-0.00230	2.9880	0.73910	-0.06558	2.9880	0.73911	-0.11148	2.9880
50.00	0.74585	-0.00223	2.6728	0.74586	-0.06358	2.6728	0.74587	-0.10807	2.6728
52.00	0.75201	-0.00216	2.3289	0.75202	-0.06177	2.3289	0.75203	-0.10500	2.3289
54.00	0.75763	-0.00211	1.9542	0.75763	-0.06014	1.9542	0.75764	-0.10223	1.9542
56.00	0.76274	-0.00205	1.5463	0.76275	-0.05867	1.5463	0.76276	-0.09974	1.5463
58.00	0.76740	-0.00201	1.1022	0.76741	-0.05734	1.1022	0.76741	-0.09748	1.1022
60.00	0.77164	-0.00197	0.6182	0.77164	-0.05615	0.6182	0.77165	-0.09544	0.6182
62.00	0.77549	-0.00193	0.0899	0.77549	-0.05507	0.0898	0.77550	-0.09361	0.0898
64.00	0.77897	-0.00189	-0.4884	0.77898	-0.05409	-0.4885	0.77899	-0.09195	-0.4885
66.00	0.78212	-0.00186	-1.1237	0.78213	-0.05322	-1.1237	0.78213	-0.09047	-1.1237
68.00	0.78495	-0.00184	-1.8246	0.78496	-0.05244	-1.8247	0.78496	-0.08914	-1.8247
70.00	0.78749	-0.00181	-2.6028	0.78749	-0.05174	-2.6028	0.78749	-0.08796	-2.6029
72.00	0.78973	-0.00179	-3.4733	0.78973	-0.05113	-3.4733	0.78974	-0.08692	-3.4733
74.00	0.79171	-0.00177	-4.4566	0.79171	-0.05059	-4.4566	0.79172	-0.08600	-4.4566
76.00	0.79343	-0.00175	-5.5815	0.79343	-0.05012	-5.5815	0.79344	-0.08520	-5.5816
78.00	0.79490	-0.00174	-6.8903	0.79490	-0.04973	-6.8903	0.79491	-0.08453	-6.8904
80.00	0.79613	-0.00173	-8.4485	0.79613	-0.04939	-8.4486	0.79614	-0.08396	-8.4486
82.00	0.79713	-0.00172	-10.3660	0.79713	-0.04912	-10.3660	0.79714	-0.08350	-10.3661
84.00	0.79790	-0.00171	-12.8487	0.79790	-0.04892	-12.8487	0.79791	-0.08315	-12.8488
86.00	0.79844	-0.00171	-16.3591	0.79845	-0.04877	-16.3591	0.79845	-0.08290	-16.3591
88.00	0.79877	-0.00170	-22.3728	0.79877	-0.04868	-22.3728	0.79878	-0.08275	-22.3728
90.00	0.79888	-0.00170	-140.4169	0.79888	-0.04865	-140.4168	0.79889	-0.08273	-140.4169

Table 9. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Ice (Fresh Water, -1°C)

CASE(8): ICE (fresh water, -1.0 deg C) (Relative Dielectric Constant = 3.0, Conductivity is defined below)

Grazing ang. deg	FREQUENCY = 6.0 MHz. COND = 0.000070 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. COND = 0.000090 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. COND = 0.000100 S/m Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.96367	-179.96271	-22.4680	0.96365	-179.98088	-22.4655	0.96365	-179.98940	-22.4647
1.00	0.92864	-179.92532	-16.6063	0.92861	-179.96172	-16.6031	0.92861	-179.97874	-16.6023
1.50	0.89485	-179.88776	-13.2411	0.89481	-179.94246	-13.2379	0.89480	-179.96805	-13.2371
2.00	0.86224	-179.84990	-10.8973	0.86219	-179.92303	-10.8942	0.86218	-179.95728	-10.8934
2.50	0.83076	-179.81165	-9.1126	0.83070	-179.90343	-9.1096	0.83069	-179.94640	-9.1088
3.00	0.80035	-179.77292	-7.6811	0.80029	-179.88356	-7.6781	0.80028	-179.93536	-7.6774
3.50	0.77097	-179.73360	-6.4930	0.77090	-179.86340	-6.4900	0.77089	-179.92416	-6.4893
4.00	0.74257	-179.69359	-5.4826	0.74250	-179.84286	-5.4797	0.74248	-179.91277	-5.4790
5.00	0.68854	-179.61104	-3.8397	0.68846	-179.80054	-3.8369	0.68844	-179.88928	-3.8362
6.00	0.63795	-179.52434	-2.5469	0.63786	-179.75606	-2.5442	0.63783	-179.86458	-2.5436
7.00	0.59051	-179.43245	-1.4947	0.59041	-179.70892	-1.4921	0.59038	-179.83841	-1.4915
8.00	0.54597	-179.33418	-0.6178	0.54586	-179.65848	-0.6153	0.54583	-179.81039	-0.6147
9.00	0.50411	-179.22823	0.1258	0.50399	-179.60410	0.1281	0.50396	-179.78020	0.1287
10.00	0.46471	-179.11299	0.7645	0.46458	-179.54495	0.7667	0.46455	-179.74736	0.7673
12.00	0.39260	-178.84702	1.8035	0.39246	-179.40837	1.8055	0.39243	-179.67151	1.8060
14.00	0.32876	-178.51648	2.6070	0.32821	-179.23857	2.6088	0.32817	-179.57721	2.6092
16.00	0.27094	-178.09099	3.2386	0.27077	-179.01988	3.2402	0.27073	-179.45573	3.2406
18.00	0.21946	-177.52065	3.7390	0.21927	-178.72661	3.7404	0.21923	-179.29282	3.7407
20.00	0.17318	-176.71660	4.1355	0.17296	-178.31202	4.1366	0.17291	-179.06290	4.1369
22.00	0.13151	-175.50250	4.4473	0.13122	-177.68715	4.4482	0.13115	-178.71512	4.4484
24.00	0.09394	-173.47154	4.6882	0.09352	-176.63725	4.6889	0.09342	-178.13107	4.6891
26.00	0.06014	-169.43294	4.8685	0.05944	-174.53069	4.8690	0.05927	-176.95660	4.8691
28.00	0.03029	-157.97131	4.9959	0.02875	-168.28699	4.9963	0.02836	-173.43538	4.9963
30.00	0.01168	-89.32300	5.0764	0.00600	-89.65504	5.0765	0.00333	-89.80935	5.0766
32.00	0.02847	-24.89236	5.1145	0.02642	-13.47878	5.1144	0.02589	-7.59295	5.1144
34.00	0.05074	-13.99928	5.1135	0.04949	-7.32617	5.1133	0.04917	-4.08820	5.1133
36.00	0.07171	-10.08473	5.0763	0.07073	-5.27001	5.0759	0.07048	-2.91726	5.0758
38.00	0.09103	-8.10085	5.0048	0.09017	-4.19568	5.0042	0.08996	-2.33506	5.0041
40.00	0.10877	-6.91156	4.9003	0.10797	-3.57487	4.8996	0.10777	-1.98888	4.8994
42.00	0.12502	-6.12448	4.7638	0.12426	-3.16545	4.7630	0.12407	-1.76077	4.7628
44.00	0.13992	-5.56867	4.5958	0.13918	-2.87695	4.5948	0.13900	-1.60012	4.5946
46.00	0.15358	-5.15785	4.3963	0.15285	-2.66402	4.3952	0.15266	-1.48160	4.3950
48.00	0.16610	-4.84382	4.1651	0.16537	-2.50142	4.1639	0.16519	-1.39111	4.1636
50.00	0.17757	-4.59753	3.9015	0.17684	-2.37400	3.9007	0.17665	-1.32022	3.8998
52.00	0.18807	-4.40046	3.6043	0.18734	-2.27210	3.6029	0.18715	-1.26353	3.6025
54.00	0.19769	-4.24023	3.2720	0.19694	-2.18929	3.2704	0.19676	-1.21747	3.2701
56.00	0.20648	-4.10828	2.9025	0.20573	-2.12113	2.9009	0.20554	-1.17956	2.9004
58.00	0.21451	-3.99848	2.4931	0.21375	-2.06442	2.4914	0.21356	-1.14802	2.4910
60.00	0.22183	-3.90636	2.0406	0.22106	-2.01686	2.0388	0.22087	-1.12158	2.0384
62.00	0.22849	-3.82857	1.5407	0.22772	-1.97671	1.5388	0.22752	-1.09925	1.5383
64.00	0.23453	-3.76255	0.9880	0.23375	-1.94265	0.9861	0.23355	-1.08031	0.9856
66.00	0.23998	-3.70635	0.3758	0.23920	-1.91365	0.3738	0.23900	-1.06418	0.3733
68.00	0.24490	-3.65841	-0.3046	0.24410	-1.88892	-0.3067	0.24390	-1.05043	-0.3072
70.00	0.24929	-3.61751	-1.0645	0.24849	-1.86783	-1.0665	0.24829	-1.03871	-1.0671
72.00	0.25319	-3.58270	-1.9187	0.25239	-1.84987	-1.9209	0.25219	-1.02873	-1.9214
74.00	0.25663	-3.55319	-2.8878	0.25581	-1.83465	-2.8900	0.25561	-1.02027	-2.8905
76.00	0.25961	-3.52835	-4.0005	0.25880	-1.82185	-4.0027	0.25859	-1.01315	-4.0032
78.00	0.26217	-3.50769	-5.2988	0.26135	-1.81120	-5.3010	0.26114	-1.00723	-5.3016
80.00	0.26430	-3.49081	-6.8482	0.26348	-1.80249	-6.8505	0.26327	-1.00239	-6.8510
82.00	0.26604	-3.47739	-8.7586	0.26521	-1.79557	-8.7609	0.26500	-0.99854	-8.7615
84.00	0.26737	-3.46718	-11.2359	0.26654	-1.79031	-11.2381	0.26634	-0.99562	-11.2387
86.00	0.26832	-3.46002	-14.7423	0.26749	-1.78662	-14.7446	0.26729	-0.99357	-14.7452
88.00	0.26889	-3.45577	-20.7538	0.26806	-1.78443	-20.7561	0.26785	-0.99235	-20.7566
90.00	0.26908	-3.45436	-138.7970	0.26825	-1.78371	-138.7993	0.26804	-0.99195	-138.7999

Table 10. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Ice (Fresh Water, -10°C)

CASE(9): ICE (fresh water, -10.0 deg C) (Relative Dielectric Constant = 3.0, Conductivity is defined below)

Grazing ang. deg	FREQUENCY = 6.0 MHz. COND = 0.000020 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. COND = 0.000027 S/m Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. COND = 0.000035 S/m Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.96365	-179.98940	-22.4647	0.96365	-179.99426	-22.4644	0.96365	-179.99629	-22.4643
1.00	0.92861	-179.97874	-16.6023	0.92861	-179.98853	-16.6020	0.92860	-179.99257	-16.6020
1.50	0.89480	-179.96805	-13.2371	0.89480	-179.98276	-13.2368	0.89480	-179.98883	-13.2368
2.00	0.86218	-179.95728	-10.8934	0.86218	-179.97693	-10.8931	0.86218	-179.98505	-10.8931
2.50	0.83069	-179.94640	-9.1088	0.83069	-179.97105	-9.1085	0.83069	-179.98125	-9.1085
3.00	0.80028	-179.93536	-7.6774	0.80027	-179.96510	-7.6771	0.80027	-179.97739	-7.6771
3.50	0.77089	-179.92416	-6.4893	0.77088	-179.95906	-6.4890	0.77088	-179.97346	-6.4890
4.00	0.74248	-179.91277	-5.4790	0.74247	-179.95293	-5.4787	0.74247	-179.96948	-5.4787
5.00	0.68844	-179.88928	-3.8362	0.68843	-179.94023	-3.8360	0.68843	-179.96127	-3.8360
6.00	0.63783	-179.86458	-2.5436	0.63782	-179.92690	-2.5434	0.63782	-179.95262	-2.5433
7.00	0.59038	-179.83841	-1.4915	0.59037	-179.91275	-1.4913	0.59037	-179.94345	-1.4912
8.00	0.54583	-179.81039	-0.6147	0.54582	-179.89763	-0.6145	0.54582	-179.93365	-0.6145
9.00	0.50396	-179.78020	0.1287	0.50395	-179.88133	0.1289	0.50395	-179.92310	0.1289
10.00	0.46455	-179.74736	0.7673	0.46454	-179.86360	0.7675	0.46454	-179.91159	0.7675
12.00	0.39243	-179.67151	1.8060	0.39242	-179.82265	1.8062	0.39241	-179.88506	1.8062
14.00	0.32817	-179.57721	2.6092	0.32816	-179.77173	2.6093	0.32816	-179.85205	2.6094
[illegible]	0.27073	-179.45573	3.2406	0.27072	-179.70613	3.2407	0.27072	-179.80954	3.2407
[illegible]	[illegible]1923	-179.29282	3.7407	0.21921	-179.61816	3.7408	0.21921	-179.75252	3.7408
[illegible]	[illegible]	-179.06290	4.1369	0.17289	-179.49400	4.1370	0.17288	-179.67204	4.1370
[illegible]	[illegible]	[illegible]178.71512	4.4484	0.13112	-179.30617	4.4485	0.13112	-179.55029	4.4485
[illegible]	[illegible]	[illegible]78.13107	4.6891	0.09338	-178.99066	4.6891	0.09338	-179.34578	4.6892
[illegible]	[illegible]	[illegible]76.95660	4.8691	0.05921	-178.35573	4.8692	0.05920	-178.93414	4.8692
[illegible]	[illegible]	-173.43538	4.9963	0.02823	-176.44508	4.9964	0.02820	-177.69432	4.9964
[illegible]	[illegible]133	-89.80935	5.0766	0.00180	-89.89668	5.0766	0.00117	-89.93313	5.0766
[illegible]	[illegible]189	-7.59295	5.1144	0.02572	-4.11866	5.1144	0.02568	-2.67239	5.1144
[illegible]	[illegible]4917	-4.08820	5.1133	0.04907	-2.21075	5.1132	0.04905	-1.43338	5.1132
[illegible]	[illegible]07346	-2.91726	5.0758	0.07040	-1.57657	5.0758	0.07038	-1.02204	5.0758
[illegible]	0.08996	-2.33506	5.0041	0.08989	-1.26164	5.0040	0.08987	-0.81784	5.0040
40.00	0.10777	-1.98888	4.8994	0.10771	-1.07448	4.8994	0.10769	-0.69650	4.8994
42.00	0.12407	-1.76077	4.7628	0.12401	-0.95119	4.7627	0.12400	-0.61657	4.7627
44.00	0.13900	-1.60012	4.5946	0.13894	-0.86438	4.5945	0.13892	-0.56029	4.5945
46.00	0.15266	-1.48160	4.3950	0.15261	-0.80033	4.3949	0.15259	-0.51878	4.3949
48.00	0.16519	-1.39111	4.1636	0.16513	-0.75145	4.1635	0.16511	-0.48709	4.1635
50.00	0.17665	-1.32022	3.8998	0.17659	-0.71314	3.8997	0.17658	-0.46226	3.8997
52.00	0.18715	-1.26353	3.6025	0.18709	-0.68252	3.6024	0.18708	-0.44241	3.6024
54.00	0.19676	-1.21747	3.2701	0.19670	-0.65764	3.2699	0.19669	-0.42628	3.2699
56.00	0.20554	-1.17956	2.9004	0.20548	-0.63716	2.9003	0.20547	-0.41300	2.9003
58.00	0.21356	-1.14802	2.4910	0.21350	-0.62012	2.4909	0.21349	-0.40196	2.4908
60.00	0.22087	-1.12158	2.0384	0.22081	-0.60584	2.0382	0.22080	-0.39270	2.0382
62.00	0.22752	-1.09925	1.5383	0.22746	-0.59378	1.5382	0.22745	-0.38488	1.5382
64.00	0.23355	-1.08031	0.9856	0.23349	-0.58355	0.9854	0.23348	-0.37825	0.9854
66.00	0.23900	-1.06418	0.3733	0.23894	-0.57484	0.3731	0.23893	-0.37261	0.3731
68.00	0.24390	-1.05043	-0.3072	0.24384	-0.56741	-0.3073	0.24383	-0.36779	-0.3074
70.00	0.24829	-1.03871	-1.0671	0.24823	-0.56108	-1.0672	0.24821	-0.36369	-1.0673
72.00	0.25219	-1.02873	-1.9214	0.25212	-0.55569	-1.9216	0.25211	-0.36019	-1.9216
74.00	0.25561	-1.02027	-2.8905	0.25555	-0.55112	-2.8907	0.25553	-0.35723	-2.8907
76.00	0.25859	-1.01315	-4.0032	0.25853	-0.54727	-4.0034	0.25851	-0.35474	-4.0034
78.00	0.26114	-1.00723	-5.3016	0.26108	-0.54407	-5.3017	0.26106	-0.35267	-5.3018
80.00	0.26327	-1.00239	-6.8510	0.26321	-0.54146	-6.8512	0.26319	-0.35097	-6.8513
82.00	0.26500	-0.99854	-8.7615	0.26494	-0.53938	-8.7616	0.26492	-0.34963	-8.7617
84.00	0.26634	-0.99562	-11.2387	0.26627	-0.53781	-11.2389	0.26626	-0.34860	-11.2390
86.00	0.26729	-0.99357	-14.7452	0.26722	-0.53670	-14.7454	0.26721	-0.34789	-14.7454
88.00	0.26785	-0.99235	-20.7566	0.26779	-0.53604	-20.7568	0.26777	-0.34746	-20.7569
90.00	0.26804	-0.99195	-138.7999	0.26798	-0.53582	-138.8000	0.26796	-0.34732	-138.8001

Table 11. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Average Land (TCI)

CASE(10): Average Land (TCI) (Relative Dielectric Constant = 10.0, Conductivity = 0.005000 S/m)

Grazing ang. deg	FREQUENCY = 6.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 15.0 MHz. Fresnel Reflect Coef		Directive gain	FREQUENCY = 30.0 MHz. Fresnel Reflect Coef		Directive gain
PSI(deg)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)	|Rv|	ang Rv(deg)	D(dBi)
0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.93505	-178.05513	-19.4723	0.94173	-179.13420	-20.3518	0.94302	-179.55827	-20.5304
1.00	0.87429	-176.09833	-13.7339	0.88670	-178.26404	-14.5827	0.88920	-179.11427	-14.7554
1.50	0.81743	-174.11749	-10.4882	0.83492	-177.38460	-11.3066	0.83830	-178.66573	-11.4735
2.00	0.76421	-172.10027	-8.2597	0.78580	-176.49123	-9.0481	0.79010	-178.21027	-9.2093
2.50	0.71440	-170.03392	-6.5860	0.73948	-175.57893	-7.3481	0.74440	-177.74542	-7.5006
3.00	0.66777	-167.90511	-5.2614	0.69554	-174.64244	-5.9917	0.70102	-177.26852	-6.1417
3.50	0.62414	-165.69902	-4.1782	0.65387	-173.67803	-4.8704	0.65979	-176.77678	-5.0229
4.00	0.58333	-163.40370	-3.2050	0.61434	-172.67355	-3.9398	0.62056	-176.26701	-4.0790
5.00	0.50964	-158.47557	-1.8098	0.54113	-170.53241	-2.4318	0.54760	-175.17927	-2.5607
6.00	0.44575	-152.98874	-0.6900	0.47500	-168.15450	-1.2832	0.48121	-173.97253	-1.3823
7.00	0.39101	-146.80544	0.2000	0.41519	-165.45786	-0.3257	0.42062	-172.60318	-0.4355
8.00	0.34900	-139.79924	0.9768	0.36112	-162.33328	0.4452	0.36510	-171.01222	0.3442
9.00	0.30754	-131.88992	1.5307	0.31237	-158.63628	1.0906	0.31438	-169.11600	0.9979
10.00	0.27851	-123.09735	2.0395	0.26868	-154.17004	1.6383	0.26788	-166.79120	1.5535
12.00	0.24497	-103.74401	2.8444	0.19043	-141.70980	2.5141	0.18822	-159.88421	2.4439
14.00	0.23579	-84.01412	3.4438	0.14680	-122.48483	3.1702	0.12002	-147.09218	3.1189
16.00	0.25380	-69.17654	3.8968	0.12540	-95.99305	3.6849	0.07512	-118.64124	3.6392
18.00	0.27881	-57.44173	4.2404	0.13753	-70.38427	4.0779	0.06937	-77.47150	4.0425
20.00	0.30654	-48.85301	4.4987	0.15662	-52.47951	4.3803	0.09710	-42.83662	4.3542
22.00	0.33472	-42.49261	4.6803	0.18687	-41.24463	4.6094	0.13377	-29.57467	4.5916
24.00	0.36249	-37.66728	4.8210	0.21882	-33.99930	4.7775	0.16893	-22.87780	4.7671
26.00	0.38867	-33.91117	4.9051	0.24800	-29.08930	4.8935	0.20227	-18.59805	4.8899
28.00	0.41302	-30.91780	4.9470	0.27810	-25.51091	4.9641	0.23289	-15.90747	4.9665
30.00	0.43551	-28.48337	4.9511	0.30213	-22.85126	4.9942	0.26088	-14.00249	5.0021
32.00	0.45620	-26.46890	4.9200	0.32611	-20.78901	4.9875	0.28642	-12.58416	5.0004
34.00	0.47522	-24.77750	4.8589	0.34816	-19.14027	4.9469	0.30975	-11.48799	4.9643
36.00	0.49268	-23.33980	4.7669	0.36839	-17.80008	4.8744	0.33106	-10.61619	4.8958
38.00	0.50877	-22.10499	4.6481	0.38690	-16.70124	4.7714	0.35055	-9.90712	4.7965
40.00	0.52344	-21.03506	4.4975	0.40408	-15.77021	4.6389	0.36840	-9.32002	4.6673
42.00	0.53697	-20.10110	4.3212	0.41973	-14.97830	4.4774	0.38478	-8.82605	4.5089
44.00	0.54940	-19.28065	4.1174	0.43418	-14.29832	4.2871	0.39977	-8.40763	4.3213
46.00	0.56083	-18.55608	3.8857	0.44740	-13.70830	4.0677	0.41354	-8.04770	4.1044
48.00	0.57133	-17.91336	3.6255	0.45950	-13.19591	3.8180	0.42619	-7.73644	3.8577
50.00	0.58099	-17.34114	3.3356	0.47079	-12.74587	3.5392	0.43780	-7.46525	3.5803
52.00	0.58987	-16.83019	3.0148	0.48109	-12.34905	3.2279	0.44848	-7.22774	3.2708
54.00	0.59803	-16.37293	2.6612	0.49084	-11.99839	2.8829	0.45824	-7.01882	2.9275
56.00	0.60551	-15.96300	2.2728	0.49922	-11.68740	2.5021	0.46721	-6.83443	2.5482
58.00	0.61236	-15.59532	1.8459	0.50717	-11.41120	2.0826	0.47542	-6.67128	2.1302
60.00	0.61864	-15.26530	1.3779	0.51445	-11.16645	1.6211	0.48297	-6.52685	1.6700
62.00	0.62438	-14.96926	0.8640	0.52108	-10.94874	1.1132	0.48977	-6.39837	1.1632
64.00	0.62957	-14.70405	0.2988	0.52712	-10.75221	0.5534	0.49599	-6.28460	0.6044
66.00	0.63429	-14.46695	-0.3746	0.53260	-10.57842	-0.0681	0.50163	-6.18382	-0.0131
68.00	0.63855	-14.25549	-1.0149	0.53754	-10.42634	-0.7510	0.50672	-6.09474	-0.6982
70.00	0.64238	-14.06828	-1.7838	0.54197	-10.29128	-1.5157	0.51128	-6.01830	-1.4622
72.00	0.64578	-13.90307	-2.6456	0.54591	-10.17272	-2.3743	0.51534	-5.94780	-2.3201
74.00	0.64879	-13.75864	-3.6214	0.54939	-10.06951	-3.3470	0.51891	-5.88708	-3.2923
76.00	0.65140	-13.63378	-4.7399	0.55242	-9.98050	-4.4628	0.52203	-5.83650	-4.4076
78.00	0.65365	-13.52742	-6.0431	0.55502	-9.90511	-5.7638	0.52470	-5.79295	-5.7082
80.00	0.65553	-13.43870	-7.5967	0.55720	-9.84237	-7.3155	0.52694	-5.75678	-7.2586
82.00	0.65705	-13.36717	-9.5104	0.55897	-9.79177	-9.2277	0.52875	-5.72703	-9.1714
84.00	0.65823	-13.31202	-11.9907	0.56033	-9.75287	-11.7083	0.53016	-5.70529	-11.6498
86.00	0.65907	-13.27291	-15.4988	0.56131	-9.72533	-15.2137	0.53115	-5.68940	-15.1571
88.00	0.65957	-13.24957	-21.5110	0.56189	-9.70890	-21.2287	0.53175	-5.67996	-21.1690
90.00	0.65974	-13.24180	-139.5545	0.56208	-9.70344	-139.2681	0.53198	-5.67682	-139.2123

IL2643

Table 12. Argument α. Very Dry Ground ($\varepsilon_r = 3.0, \sigma = 1.0 \times 10^{-4}$ S/m)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) = arctan [(1)/(2)]	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	−π	−π	−π	0.000	0.000	0.000	0.000	−π/2	−π/2
5	0.689	−3.132	−3.132	−3.191	−0.007	0.034	0.311	0.312	−0.023	0.109
10	0.465	−3.119	−3.119	−3.157	−0.011	0.007	0.535	0.535	−0.021	0.109
16	0.271	−3.094	−3.094	−3.281	−0.013	0.038	0.729	0.732	−0.018	0.052
20	0.173	−3.060	−3.060	−3.292	−0.014	0.026	0.828	0.829	−0.017	0.031
26	0.061	−2.850	−2.880	−3.177	−0.016	−0.002	0.942	0.939	−0.017	−0.002
30	0.017	−1.554	−1.554	−1.893	−0.017	−0.016	1.000	0.995	−0.017	−0.016

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) = arctan [(1)/(2)]	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	−π	−π	−π	0.000	0.000	0.000	0.000	−π/2	−π/2
5	0.689	−3.140	−3.140	−3.436	−0.001	0.200	0.311	0.341	−0.003	0.530
10	0.465	−3.137	−3.137	−3.726	−0.002	0.257	0.535	0.612	−0.004	0.398
16	0.271	−3.132	−3.132	−4.067	−0.003	0.216	0.729	0.837	−0.004	0.253
20	0.173	−3.125	−3.125	−4.285	−0.003	0.157	0.827	0.928	−0.004	0.168
26	0.059	−3.088	−3.088	−4.575	−0.003	0.058	0.941	0.992	−0.003	0.058
30	0.003	−1.567	−1.567	−3.263	−0.003	0.000	0.997	0.997	−0.003	0.000

β (rad) = angle R_v (rad) − $(2\pi/\lambda_o)\, 2\, h \sin\psi$

$\alpha = \arctan [R_v \sin\beta/(1 + R_v \cos\beta)]$

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Table 13. Argument α. Medium Dry Ground ($\varepsilon_r = 15.0, \sigma = 1.0 \times 10^{-4}$ S / m)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) $= \arctan[(1)/(2)]$	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	π	π	π	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.481	-3.068	-3.068	-3.127	-0.035	-0.007	0.520	0.519	-0.067	-0.013
10	0.182	-2.893	-2.893	-3.011	-0.045	-0.024	0.824	0.820	-0.055	-0.029
16	0.070	-0.717	-0.717	-0.904	-0.046	-0.055	1.053	1.043	-0.044	-0.053
20	0.165	-0.277	-0.277	-0.509	-0.045	-0.080	1.159	1.144	-0.039	-0.070
26	0.279	-0.155	-0.155	-0.452	-0.043	-0.126	1.276	1.251	-0.034	-0.100
30	0.338	-0.124	-0.124	-0.463	-0.042	-0.151	1.335	1.334	-0.031	-0.113

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) $= \arctan[(1)/(2)]$	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	π	π	π	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.482	-3.127	-3.127	-3.423	-0.026	-0.134	0.518	0.537	-0.050	0.245
10	0.180	-3.091	-3.091	-3.680	-0.010	-0.092	0.820	0.845	-0.012	0.108
16	0.050	-0.189	-0.189	-1.124	-0.000	-0.045	1.049	1.022	0.000	0.044
20	0.155	-0.059	-0.059	-3.452	-0.000	-0.047	1.155	0.852	0.000	0.055
26	0.272	-0.032	-0.032	-1.519	-0.000	-0.272	1.272	1.014	0.000	-0.268
30	0.331	-0.026	-0.026	-1.727	-0.000	-0.327	1.331	0.950	0.000	-0.332

β (rad) = angle R_v (rad) $- (2\pi/\lambda_o)\, 2\, h \sin\psi$

$\alpha = \arctan[R_v \sin\beta/(1 + R_v \cos\beta)]$

IL2645

Table 14. Argument α. Wet Ground ($\varepsilon_r = 30.0$, $\sigma = 1.0 \times 10^{-4}$ S / m)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) $= \arctan[(1)/(2)]$	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.334	−2.573	−2.573	−2.632	−0.180	−0.163	0.719	0.708	−0.245	−0.226
10	0.205	−1.233	−1.233	−1.351	−0.193	−0.200	1.068	1.045	−0.179	−0.189
16	0.347	−0.539	−0.539	−0.726	−0.178	−0.230	1.298	1.259	−0.136	−0.181
20	0.429	−0.396	−0.396	−0.628	−0.165	−0.252	1.396	1.347	−0.118	−0.185
26	0.519	−0.290	−0.290	−0.587	−0.148	−0.287	1.497	1.432	−0.099	−0.198
30	0.564	−0.248	−0.248	−0.587	−0.138	−0.312	1.547	1.470	−0.089	−0.209

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\lvert R_v \rvert$	Angle R_v (rad)	β (rad)		(1) $\lvert R_v \rvert \sin\beta$		(2) $1 + \lvert R_v \rvert \cos\beta$		α (rad) $= \arctan[(1)/(2)]$	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.346	−3.019	−3.019	−3.315	−0.042	−0.060	0.659	0.659	−0.064	−0.091
10	0.049	−1.822	−1.822	−2.411	−0.047	−0.033	1.988	0.964	−0.048	−0.034
16	0.220	−0.209	−0.209	−1.144	−0.046	−0.200	1.215	1.091	−0.038	−0.181
20	0.318	−0.136	−0.136	−1.296	−0.043	−0.306	1.315	1.086	−0.033	−0.275
26	0.424	−0.093	−0.093	−1.580	−0.039	−0.424	1.422	0.996	−0.027	−0.402
30	0.476	−0.079	−0.079	−1.775	−0.003	−0.466	1.475	0.903	−0.002	−0.476

β (rad) = angle R_v (rad) − $(2\pi/\lambda_o)\, 2\, h \sin\psi$

$\alpha = \arctan[R_v \sin\beta/(1 + R_v \cos\beta)]$

and wet ground, respectively. Numerical values are given at 6 MHz for $h/\lambda_o = 0, 0.054$ and at 30 MHz for $\lambda_o = 0, 0.270$. The normalized heights $h/\lambda_o = 0, 0.054$ and 0.270 correspond to the mid-point at 6 MHz and 30 MHz, respectively, of a vertical monopole of length $\ell = 5.4$ m.

The mean argument $\overline{\alpha}(\theta)$, averaged over the m elements of the array for a plane wave incident from the direction θ, is given by

$$\overline{\alpha}(\theta) = (1/m) \sum_{k=1}^{m} \alpha_k(\theta) \qquad (2\text{-}10)$$

The variability of the argument α_k from element to element causes an array RMS phase error α_{kms} and beam pointing errors B_θ, B_ϕ. The RMS phase error, at the diffraction focus of the array [3], is given by

$$\alpha_{rms}(\theta) = \left\{ \left(\sum_{k=1}^{m} (1/m)[\alpha_k(\theta) - \overline{\alpha}(\theta)]^2 \right) - \alpha_B^2(\theta) \right\}^{1/2} \qquad (2\text{-}11)$$

where $\alpha_B(\theta)$ is the RMS phase error caused by a linear phase shift resulting from beam pointing errors. The rms phase error $\alpha_B(\theta)$ is given by equation (A-13) in the appendix as

$$\alpha_B = \left\{ \frac{(1/m)\left[\sum_{k=1}^{m}(x_k/r_A)(\alpha_k - \overline{\alpha})\right]^2}{\sum_{k=1}^{m}(x_k/r_A)^2} + \frac{(1/m)\left[\sum_{k=1}^{m}(y_k/r_A)(\alpha_k - \overline{\alpha})\right]^2}{\sum_{k=1}^{m}(y_k/r_A)^2} \right\}^{1/2}$$

., are uncorrelated (2-12)

where

x_k, y_k = projections of the kth element location onto the x and y axes, respectively, whose origin is at the center of the array.

The non-homogeneous earth causes beam pointing errors B_θ and B_ϕ, in the elevation and azimuthal directions, respectively. The beam pointing errors, for a plane wave incident from the true direction (θ,ϕ) are given by equations (A-20) and (A-21) as

$$B_\theta = \theta_{apparent} - \theta = -(1/\cos\theta)[(\hat{u}-u)\cos\phi + (\hat{v}-v)\sin\phi],\ B_\theta << 1 \text{ rad} \tag{2-13}$$

$$B_\phi = \phi_{apparent} - \phi = -(1/\sin\theta)[(\hat{u}-u)\sin\phi + (\hat{v}-v)\cos\phi],\ B_\phi << 1 \text{ rad} \tag{2-14}$$

where $(\theta_{apparent}, \phi_{apparent})$ is the apparent beam direction when the beam has a true direction (θ,ϕ). The quantities $(\hat{u} - u)$ and $(\hat{v} - v)$ are given by equations (A-9) and (A-10), respectively, as

$$\hat{u} - u = (\lambda_o / 2\pi r_A) \sum_{k=1}^{m} (x_k / r_A)(\alpha_k - \overline{\alpha}) / \sum_{k=1}^{m} (x_k / r_A)^2 \tag{2-15}$$

$$\hat{v} - v = (\lambda_o / 2\pi r_A) \sum_{k=1}^{m} (y_k / r_A)(\alpha_k - \overline{\alpha}) / \sum_{k=1}^{m} (y_k / r_A)^2 \tag{2-16}$$

where x_k, y_k are assumed to be uncorrelated.

The parameters α_B, B_θ, B_ϕ, $\hat{u} - u$, and $\hat{v}$ - are zero for the cases: (1) homogeneous earth $(\alpha_k = \overline{\alpha})$ and (2) non-homogeneous earth with randomly-distributed non-homogenities and elements $\left(\sum x_k\ (\alpha_k - \overline{\alpha}) = \sum y_k\ (\alpha_k - \overline{\alpha}) = 0\right)$. However, for non-homogeneous earth with systematically-distributed non-homogeneities or elements $\left(\sum x_k\ (\alpha_k - \overline{\alpha}) \neq 0),\ \sum y_k\ (\alpha_k - \overline{\alpha}) \neq 0),\ \right)$, then α_B, B_θ,B_ϕ, $\hat{u} - u$ and $\hat{v} - v$ are non-zero.

SECTION 3

NUMERICAL RESULTS

Numerical results for the RMS phase error α_{rms} and boresight errors B_θ, B_ϕ, are given in this section for cases where a fraction of the elements are located in proximity to earth of permittivity ε_I^* and the remaining fraction are in proximity to earth of permittivity ε_{II}^*. This type of non-homogeneous earth is designated as "two-level" non-homogeneous earth. Accordingly, the parameters ε_k^*, γ_k, β_k, and $R_{k,v}$ at the kth element are given by

$$\varepsilon_{ku}^*, \alpha_k, \beta_k, R_{k,v} \doteq \begin{cases} \varepsilon_{\mathrm{I}}^*, \alpha_I, R_{I,V} \text{ at p elements,} \\ \qquad \text{two - level non - homogeneous earth} \\ \varepsilon_{\mathrm{II}}^*, \alpha_{II}, R_{II,V} \text{ at m - p elements} \end{cases} \tag{3-1}$$

Furthermore, it is assumed that the elements are randomly distributed over the circular area of radius r_A and that projections x_k and y_k of the kth element onto the x and y axes, respectively, are uncorrelated.

3.1 Randomly-Distributed Two-Level Non-Homogeneous Earth

Consider first the case the two permittivities ε_I^* and ε_{II}^* are randomly distributed with occurrence frequencies p/m for ε_I^* and 1 - (p/m) for ε_{II}^*. Consequently,

$$\sum_{k=1}^{m} x_k \alpha_k = \sum_{k=1}^{m} y_k \alpha_k = 0, \text{ randomly distributed non-homogenities and elements} \tag{3-2}$$

For this case, equations (2-10) through (2-16) reduce to

$$\overline{\alpha}(\theta) = (p/m)\,\alpha_I(\theta) + [1-(p/m)]\,\alpha_{II}(\theta), \quad \text{two-level non-homogeneous earth} \tag{3-3}$$

$$\alpha_{rms}(\theta) = \left\{(p/m)[\alpha_I(\theta) - \overline{\alpha}(\theta)]^2 + [1-(p/m)][\alpha_{II}(\theta) - \overline{\alpha}(\theta)]^2\right\}^{1/2},$$

two-level non-homogeneous earth, randomly-distributed elements and non-homogenities (3-4)

$$\alpha_B = B_\theta = B_\phi = \hat{u} - u = \hat{v} - v = 0,$$

randomly distributed elements and earth non-homogenities (3-5)

Numerical values of the RMS phase error $\alpha_{rms}(\theta)$ are given in tables 15 through 17 for three compositions of two-level randomly-distributed non-homogeneous earth for p/m = 0.5 and elevation angles $60 \leq \theta \leq 90$ degees. Table 15 is for very dry ground/medium dry ground. Table 16 is for medium dry ground/wet ground. Table 17 is for very dry ground/wet ground. Numerical values are given at 6 MHz for normalized height h/λ_0= 0, 0.054 and at 30 MHz for h/λ_0= 0, 0.270.

The RMS phase errors of tables 15 through 17 are summarized in table 18. The largest phase errors occur for very dry ground/wet ground. At 6 MHz, the maximum RMS phase error is 10 degrees and occurs for θ = 85 degrees, h/λ_0= 0, 0.054. At 30 MHz, the maximum RMS phase error is 18 degrees and occurs for θ = 85 degrees, h/λ_0= 0, 0.270. The RMS phase error generally increases with increasing values of h/λ_0 (for modest value of h/λ_0). The dependence of RMS phase error upon θ is generally non-monotonic.

3.2 SYSTEMATICALLY-DISTRIBUTED TWO-LEVEL NON-HOMOGENEOUS EARTH

Consider now the case where the two-permittivities ε_I^* and ε_{II}^* are <u>not</u> randomly distributed over the array area of radius r_A. In particular consider the case where the left half of the array of p = m/2 elements is over earth with a permittivity ε_I^* and the right half of the array of m - p = m/2 elements is over earth with a permittivity ε_{II}^*. Accordingly, p/m = 0.5 and

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Table 15. RMS Phase Error α_{rms} (Very Dry Ground/Medium Dry Ground, p/m = 0.5, Randomly-Distributed Non-Homogenities)

6 MHz ($\lambda_o = 50m$)

Grazing Angle, ψ (deg)	α_I (rad) Very Dry Ground		α_{II} (rad) Medium Dry Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.023	0.109	−0.067	−0.013	−0.045	0.048	0.022	0.065	1.26	3.72
10	−0.021	0.109	−0.055	−0.029	−0.038	0.040	0.017	0.069	0.97	3.95
16	−0.018	0.052	−0.044	−0.053	−0.040	−0.001	0.016	0.053	0.92	3.04
20	−0.017	0.031	−0.039	−0.070	−0.028	−0.020	0.011	0.051	0.63	2.92
26	0.017	−0.002	−0.034	−0.100	−0.026	−0.051	0.009	0.049	0.52	2.81
30	0.017	0.016	−0.031	−0.113	−0.024	−0.065	0.007	0.049	0.40	2.81

30 MHz ($\lambda_o = 10m$)

Grazing Angle, ψ (deg)	α_I (rad) Very Dry Ground		α_{II} (rad) Medium Dry Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.003	0.530	−0.050	0.245	−0.027	0.388	0.023	0.143	1.32	8.19
10	−0.004	0.398	−0.012	0.108	−0.008	0.253	0.004	0.145	0.23	8.31
16	−0.004	0.253	0.000	0.044	−0.002	0.149	0.002	0.105	0.11	6.02
20	−0.004	0.168	0.000	0.055	−0.002	0.112	0.002	0.057	0.11	3.27
26	−0.003	0.058	0.000	−0.268	−0.002	−0.105	0.002	0.163	0.11	9.33
30	−0.003	0.000	0.000	−0.332	−0.002	−0.166	0.002	0.166	0.11	9.51

$\overline{\alpha} = 0.5\,(\alpha_I + \alpha_{II})$

$\overline{\alpha}_{rms} = \{0.5\,[(\alpha_I - \overline{\alpha})^2 + (\alpha_{II} - \overline{\alpha})^2]\}^{1/2}$

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Table 16. RMS Phase Error α_{rms} (Medium Dry Ground/Wet Ground, p/m = 0.5, Randomly-Distributed Non-Homogenities)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	α_I (rad) Medium Dry Ground		α_{II} (rad) Wet Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.067	−0.013	−0.245	−0.226	−0.156	−0.120	0.089	0.168	5.10	9.64
10	−0.055	−0.029	−0.179	−0.189	−0.117	−0.109	0.062	0.080	3.52	4.58
16	−0.044	−0.053	−0.136	−0.181	−0.090	−0.117	0.046	0.064	2.63	3.67
20	−0.039	−0.070	−0.118	−0.185	−0.079	−0.128	0.040	0.057	2.29	3.27
26	−0.034	−0.100	−0.099	−0.198	−0.067	−0.149	0.033	0.052	1.89	2.98
30	−0.031	−0.113	−0.089	−0.209	−0.060	−0.161	0.029	0.051	1.66	2.92

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	α_I (rad) Medium Dry Ground		α_{II} (rad) Wet Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.050	0.245	−0.064	−0.091	−0.057	0.077	0.007	0.168	0.40	9.62
10	−0.012	0.108	−0.048	−0.034	−0.030	0.037	0.018	0.071	1.03	4.07
16	0.000	0.044	−0.038	−0.181	−0.019	−0.069	0.019	0.113	1.09	6.47
20	0.000	0.055	−0.033	−0.275	−0.017	−0.110	0.017	0.165	0.97	9.45
26	0.000	−0.268	−0.027	−0.402	−0.014	−0.335	0.017	0.067	0.80	3.83
30	0.000	−0.332	−0.002	−0.476	−0.001	−0.404	0.001	0.072	0.06	4.13

$\overline{\alpha} = 0.5\,(\alpha_I + \alpha_{II})$

$\overline{\alpha}_{rms} = \{0.5\,[(\alpha_I - \overline{\alpha})^2 + (\alpha_{II} - \overline{\alpha})^2]\}^{1/2}$

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Table 17. RMS Phase Error α_{rms} (Very Dry Ground/Wet Ground, p/m = 0.5, Randomly-Distributed Non-Homogenities)

6 MHz ($\lambda_o = 50m$)

Grazing Angle, ψ (deg)	α_I (rad) Very Dry Ground		α_{II} (rad) Wet Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.023	0.109	−0.245	−0.226	−0.134	−0.059	0.111	0.168	6.36	9.62
10	−0.021	0.109	−0.179	−0.189	−0.100	−0.040	0.080	0.149	4.58	8.54
16	−0.018	0.052	−0.136	−0.181	−0.077	−0.065	0.059	0.117	3.38	6.70
20	−0.017	0.031	−0.118	−0.185	−0.068	−0.077	0.051	0.108	2.92	6.19
26	−0.017	−0.002	−0.099	−0.198	−0.058	−0.100	0.041	0.098	2.35	5.61
30	−0.017	−0.016	−0.089	−0.209	−0.053	−0.113	0.036	0.097	2.06	5.56

30 MHz ($\lambda_o = 10m$)

Grazing Angle, ψ (deg)	α_I (rad) Very Dry Ground		α_{II} (rad) Wet Ground		$\overline{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	−0.003	0.530	−0.064	−0.091	−0.034	0.220	0.031	0.311	1.78	17.8
10	−0.004	0.398	−0.048	−0.034	−0.026	0.182	0.022	0.216	1.26	12.4
16	−0.004	0.253	−0.038	−0.181	−0.021	0.036	0.017	0.217	0.97	12.4
20	−0.004	0.168	−0.033	−0.275	−0.019	−0.054	0.015	0.222	0.86	12.7
26	−0.003	0.058	−0.027	−0.402	−0.015	−0.172	0.012	0.230	0.69	13.2
30	−0.003	0.000	−0.002	−0.476	−0.003	−0.238	0.001	0.238	0.57	13.6

$\overline{\alpha} = 0.5\,(\alpha_I + \alpha_{II})$

$\overline{\alpha}_{rms} = \{0.5\,[(\alpha_I - \overline{\alpha})^2 + (\alpha_{II} - \overline{\alpha})^2]\}^{1/2}$

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Table 18. Summary of RMS Phase Errors (p/m = 0.5, Randomly-Distributed Two-Level Non-Homogeneous Earth)

(a) 6 MHz (λ_o = 50m)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.4	2.8	1.7	2.9	2.1	5.6
64	0.5	2.8	1.9	3.0	2.3	5.6
70	0.6	2.9	2.3	3.3	2.9	6.2
74	0.9	3.0	2.6	3.7	3.4	6.7
80	1.0	4.0	3.5	4.6	4.6	8.5
85	1.3	3.7	5.1	9.6	6.4	9.6
90	0	0	0	0	0	0

(b) 30 MHz (λ_o = 10m)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	0.1	9.5	0.1	4.1	0.6	13.6
64	0.1	9.3	0.8	3.8	0.7	13.2
70	0.1	3.3	1.0	9.5	0.9	12.7
74	0.1	6.0	1.1	6.5	1.0	12.4
80	0.2	8.3	1.0	4.1	1.3	12.4
85	1.3	8.2	0.4	9.6	1.8	17.8
90	0	0	0	0	0	0

$$
\varepsilon^{\bullet}_{ku}, \alpha_k, \beta_k, R_{k,v} = \begin{cases} \varepsilon^{\bullet}_{\mathrm{I}}, \alpha_I, \beta_I, R_{I,V} \text{ at } (\mathrm{m}/2) \text{ elements}, -\mathrm{r_A} \le x_k \le 0 \\ \\ \varepsilon^{\bullet}_{\mathrm{II}}, \alpha_{II,v}, \beta_{II}, R_{II,v} \text{ at } (\mathrm{m}/2) \text{ elements}, \ 0 \le x_k \le r_A \end{cases} \tag{3-6}
$$

If the probability densities of the spacing of element location projections on the x and y axes were specified, then the cumulative probabilities, of the RMS phase error and beam pointing errors exceeding specified values, could be determined [4]. In this study, only the expected value of the RMS phase error and beam pointing errors are determined.

The expected value $\langle s \rangle$ of the spacing between element location projections x_k, x_{k+1} on the y axis, is given by

$$
\langle s \rangle = r_A/(m-1) \tag{3-7}
$$

For m >> 1

$$
\langle s \rangle / r_A \ll \langle |x_k| \rangle / r_A, \langle |y_k| \rangle / r_A \ll 1 \tag{3-8}
$$

Denoting the ordered projections of x_k on the negative and positive x axes as x_q then

$$
\langle (x_k)_{ordered} \rangle = \langle x_q \rangle = \begin{cases} -q \ \langle s \rangle; q = 1,2,\ldots\ldots m/2; \ -r_A \le x_q < 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad m \gg 1 \\ q \ \langle s \rangle; q = 1,2,\ldots\ldots m/2; \quad 0 \le x_q \le r_A \end{cases} \tag{3-9}
$$

For α_k and x_k given by eqs. (3-6) and (3-9), respectively,

$$
\sum_{k=1}^{m} (x_k / r_A)(\alpha_k - \overline{\alpha}) = \frac{1}{m-1}\left[(\alpha_I - \overline{\alpha})\sum_{q=1}^{m/2} -q + (\alpha_{II} - \overline{\alpha})\sum_{q=1}^{m/2} q\right]
$$

$$= \frac{1}{m-1}(\alpha_{II} - \alpha_I)\frac{(m/2)[(m/2)+1]}{2} = \frac{m(m+2)}{q(m-1)}(\alpha_{II} - \alpha_I) \qquad (3\text{-}10)$$

$$\sum_{k=1}^{m}(y_k / r_A)(\alpha_k - \overline{\alpha}) = 0;\ \text{y}_k \text{ and x}_k \text{ are randomly distributed} \qquad (3\text{-}11)$$

$$\sum_{k=1}^{m}(x_k / r_A)^2 = \sum_{k=1}^{m}(y_k / r_A)^2 = 2\sum_{q=1}^{m/2}(x_q / r_A)^2$$

$$= \frac{2}{(m-1)^2}\sum_{q=1}^{m/2} q^2 = \frac{2}{(m-1)^2}\frac{(m/2)[(m/2)+1](m+1)}{6}$$

$$= \frac{m(m+1)(m+2)}{12(m-1)^2} \qquad (3\text{-}12)$$

Substituting equations (3-10) through (3-12) into equations (2-10) through (2-16),

$$\overline{\alpha}(\theta) = (\alpha_I + \alpha_{II}) / 2 \qquad (3\text{-}13)$$

$$\alpha_{rms}(\theta) = \left\{0.5[(\alpha_I - \overline{\alpha})^2 + (\alpha_{II} - \overline{\alpha})^2] - \alpha_B^2\right\}^{1/2} \qquad (3\text{-}14)$$

$$\alpha_B = \left\{\frac{1}{m}\left[\frac{m(m+2)(\alpha_{II} - \alpha_I)}{8(m-1)}\right]^2 \frac{12(m-1)^2}{m(m+1)(m+2)}\right\}^{1/2}$$

$$= \left[\frac{3}{16}\frac{(m+2)(\alpha_{II} - \alpha_I)^2}{(m+1)}\right]^{1/2}$$

$$= [(3/16)(\text{m}+2)/(\text{m}+1)]^{1/2}|\alpha_{II} - \alpha_I| \qquad (3\text{-}15)$$

$$\hat{\text{u}} - \text{u} = \frac{\lambda_o}{2\pi r_A}\frac{m(m+2)(\alpha_{II} - \alpha_I)}{8(m-1)}\frac{12(m-1)^2}{m(m+1)(m+2)}$$

$$= \frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_{II} - \alpha_I)}{(m+1)} \tag{3-16}$$

$$\hat{v} - v = 0 \tag{3-17}$$

$$B_\theta = -\frac{1}{\cos\theta}(\hat{u} - u)\cos\phi = -\frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_{II} - \alpha_I)}{(m+1)}$$

$$\approx -(BW)_{\theta,3dB} \frac{3}{2\pi} \frac{(m-1)(\alpha_{II} - \alpha_I)\cos\phi}{(m+1)}, B_\theta << 1 rad \tag{3-18}$$

where

$(BW)_{\theta,3dB}$ = elevation beamwidth of the array main beam at its half-power points

= $(1.02\ \lambda_o/2r_A \cos\theta)$ rad (3-19)

$$B_\phi = \frac{1}{\sin\theta}(\hat{u} - u)\sin\phi = \frac{\sin\phi}{\sin\theta} \frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_{II} - \alpha_I)}{(m+1)}$$

$$\approx (BW)_{\phi,3dB} \frac{3}{2\pi} \frac{(m-1)(\alpha_{II} - \alpha_I)}{(m+1)} \frac{\sin\phi}{\sin\theta}, B_\phi << 1\ rad \tag{3-20}$$

where

$(BW)_{\theta,3dB}$ = azimuth beamwidth of the array main beam at its half-power points

= $(1.02\ \lambda_o/2r_A$ rad (3-21)

Numerical values of the RMS phase error α_{rms} and beam pointing errors B_θ, B_ϕ are given in tables 19 through 21 for systematically-distributed non-homogeneous very dry ground/medium dry ground, medium dry ground/wet ground, and very dry ground/wet ground, respectively. The RMS phase errors are approximately 50 percent less than those

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Table 19. RMS Phase Error and Beam Pointing Errors (Very Dry Ground/Medium Dry Ground, p/m = 0.5, Earth Non-Homogenities Systematically Distributed so that each Half of Array is over Different Earth, m = 96 Elements)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta/(BW)_{\theta,3\,dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi/(BW)_{\phi,3\,dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	0	0	0	0	0	0	0	0	0	0
5	−0.044	−0.122	0.008	0.023	0.020	0.061	0.021	0.057	−0.021	−0.057
10	−0.034	−0.138	0.006	0.026	0.016	0.064	0.016	0.065	−0.016	−0.066
16	−0.026	−0.105	0.005	0.020	0.015	0.049	0.012	0.049	−0.012	−0.051
20	−0.022	−0.101	0.004	0.019	0.010	0.047	0.010	0.047	−0.011	−0.050
26	−0.017	−0.098	0.003	0.019	0.008	0.045	0.008	0.046	−0.009	−0.051
30	−0.014	−0.097	0.003	0.018	0.006	0.046	0.007	0.045	−0.008	−0.052

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta/(BW)_{\theta,3\,dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi/(BW)_{\phi,3\,dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	0	0	0	0	0	0	0	0	0	0
5	−0.047	−0.285	0.009	0.054	0.021	0.132	0.022	0.133	−0.022	−0.132
10	−0.008	−0.290	0.002	0.055	0.003	0.134	0.004	0.136	−0.044	−0.138
16	0.004	−0.209	0.001	0.040	0.002	0.097	−0.002	0.098	0.002	−0.102
20	0.004	−0.113	0.001	0.021	0.002	0.053	−0.002	0.053	0.002	−0.056
26	0.003	−0.326	0.001	0.062	0.002	0.151	−0.001	0.152	0.001	−0.169
30	0.003	−0.332	0.001	0.063	0.002	0.154	−0.001	0.155	0.001	−0.179

$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2}|\alpha_{II} - \alpha_I|$, $\alpha_{rms} = (\alpha^2_{rms,\,\alpha_B = 0} - \alpha^2_B)^{1/2}$

$$\frac{B_\theta}{(BW)_{\theta,\,3\,dB}} = \frac{-3}{2\pi}\,\frac{m-1}{m+1}\,(\alpha_{II} - \alpha_I)\cos\phi,\quad \frac{B_\phi}{(BW)_{\phi,\,3\,dB}} = \frac{3}{2\pi}\,\frac{m-1}{m+1}\,\frac{\sin\phi}{\sin\theta}\,(\alpha_{II} - \alpha_I)$$

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Table 20. RMS Phase Error and Beam Pointing Errors (Medium Dry Ground/Wet Ground, p/m = 0.5, Earth Non-Homogenities Systematically Distributed so that each Half of Array is over Different Earth, m = 96 Elements)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta/(BW)_{\theta,3\,dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi/(BW)_{\phi,3\,dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	0	0	0	0	0	0	0	0	0	0
5	–0.178	–0.213	0.077	0.093	0.045	0.140	0.083	0.100	–0.083	–0.100
10	–0.124	–0.160	0.054	0.070	0.030	0.039	0.058	0.075	–0.059	–0.076
16	–0.092	–0.128	0.040	0.056	0.023	0.031	0.043	0.060	–0.045	–0.062
20	–0.079	–0.115	0.034	0.050	0.021	0.027	0.037	0.054	–0.039	–0.057
26	–0.065	–0.098	0.028	0.043	0.021	0.029	0.030	0.046	–0.033	–0.051
30	–0.058	–0.096	0.025	0.042	0.015	0.029	0.027	0.045	–0.031	–0.052

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta/(BW)_{\theta,3\,dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi/(BW)_{\phi,3\,dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	0	0	0	0	0	0	0	0	0	0
5	–0.014	–0.336	0.006	0.146	0.004	0.083	0.007	0.157	–0.007	–0.157
10	–0.036	–0.142	0.016	0.062	0.008	0.035	0.017	0.066	–0.017	–0.067
16	–0.038	–0.225	0.017	0.098	0.008	0.056	0.018	0.105	–0.018	–0.109
20	–0.033	–0.330	0.014	0.144	0.010	0.081	0.015	0.154	–0.016	–0.164
26	–0.027	–0.134	0.012	0.058	0.007	0.034	0.013	0.063	–0.014	–0.070
30	–0.002	–0.144	0.001	0.063	0	0.035	0.001	0.067	–0.001	–0.077

$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2}|\alpha_{II} - \alpha_I|$, $\alpha_{rms} = (\alpha^2_{rms,\,\alpha_B=0} - \alpha^2_B)^{1/2}$

$$\frac{B_\theta}{(BW)_{\theta,3\,dB}} = \frac{-3}{2\pi}\,\frac{m-1}{m+1}\,(\alpha_{II} - \alpha_I)\cos\phi,\quad \frac{B_\phi}{(BW)_{\phi,3\,dB}} = \frac{3}{2\pi}\,\frac{m-1}{m+1}\,\frac{\sin\phi}{\sin\theta}\,(\alpha_{II} - \alpha_I)$$

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Table 21. RMS Phase Error and Beam Pointing Errors (Very Dry Ground/Wet Ground, p/m = 0.5, Earth Non-Homogenities Systematically Distributed so that each Half of Array is over Different Earth, m = 96 Elements)

6 MHz (λ_o = 50m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta / (BW)_{\theta,3\ dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi / (BW)_{\phi,3\ dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	0	0	0	0	0	0	0	0	0	0
5	–0.222	–0.335	0.097	0.146	0.054	0.083	0.104	0.157	–0.104	–0.158
10	–0.158	–0.298	0.069	0.130	0.040	0.073	0.074	0.139	–0.075	–0.141
16	–0.118	–0.233	0.051	0.101	0.030	0.059	0.055	0.109	–0.057	–0.113
20	–0.101	–0.216	0.044	0.094	0.026	0.053	0.047	0.101	–0.050	–0.107
26	–0.082	–0.196	0.036	0.085	0.020	0.050	0.038	0.092	–0.042	–0.102
30	–0.072	–0.193	0.031	0.084	0.018	0.049	0.034	0.090	–0.039	–0.104

30 MHz (λ_o = 10m)

Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad: Elevation Pointing Error $B_\theta / (BW)_{\theta,3\ dB}$		$\phi = \pi/2$ rad: Azimuthal Pointing Error $B_\phi / (BW)_{\phi,3\ dB}$	
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	0	0	0	0	0	0	0	0	0	0
5	–0.061	–0.621	0.027	0.270	0.015	0.154	0.029	0.290	–0.029	–0.291
10	–0.044	–0.432	0.019	0.188	0.011	0.106	0.021	0.202	–0.021	–0.205
16	–0.034	–0.434	0.015	0.189	0.008	0.107	0.016	0.202	–0.017	–0.210
20	–0.029	–0.443	0.013	0.193	0.007	0.110	0.014	0.207	–0.015	–0.220
26	–0.024	0.460	0.010	0.200	0.007	0.114	0.011	–0.215	–0.012	0.239
30	–0.001	0.476	0.000	0.207	0.001	0.117	0.000	–0.223	0.000	0.248

$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2} |\alpha_{II} - \alpha_I|$, $\alpha_{rms} = (\alpha^2_{rms,\ \alpha_B = 0} - \alpha^2_B)^{1/2}$

$$\frac{B_\theta}{(BW)_{\theta,\ 3\ dB}} = \frac{-3}{2\pi}\ \frac{m-1}{m+1}\ (\alpha_{II} - \alpha_I) \cos\phi,\quad \frac{B_\phi}{(BW)_{\phi,\ 3\ dB}} = \frac{3}{2\pi}\ \frac{m-1}{m+1}\ \frac{\sin\phi}{\sin\theta}\ (\alpha_{II} - \alpha_I)$$

given in tables 15 through 17 for randomly-distributed non-homogeneous earth. The elevation and azimuthal beam pointing errors are approximately one-tenth the 3 dB beamwidths of the array main beam.

The expected values of the RMS phase error and elevation beam pointing error of tables 19 through 21 are summarized in tables 22 and 23, respectively. The largest RMS phase error is 9 deg. The largest beam pointing error is 0.3 of a beamwidth. The maximum errors occur for $\theta = 85$ deg, 30 MHz, $h/\lambda = 0.270$, and very dry ground/wet ground.

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Table 22. Summary of RMS Phase Errors (p/m = 0.5, Two-Level Non-Homogeneous Earth Systematically Distributed so that each Half of Array is over Different Earth)

(a) 6 MHz (λ_o = 50m)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.3	2.6	0.9	1.7	1.0	4.8
64	0.5	2.6	1.2	1.7	1.1	4.2
70	0.6	2.7	1.2	1.5	1.5	3.4
74	0.9	2.8	1.3	1.8	1.7	3.0
80	0.9	3.7	1.7	2.2	2.3	2.9
85	1.1	3.5	2.6	8.0	3.1	2.8
90	0	0	0	0	0	0

(b) 30 MHz (λ_o = 10m)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	0.1	8.8	0.3	4.8	0.9	6.7
64	0.1	8.7	0.5	2.0	0.6	6.5
70	0.1	3.0	0.5	3.2	0.5	6.3
74	0.1	5.6	0.6	4.6	0.4	6.3
80	0.2	7.7	0.4	1.9	0.4	6.1
85	1.2	7.5	0	2.0	0.1	8.8
90	0	0	0	0	0	0

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Table 23. Summary of Elevation Beam Pointing Errors ($p/m = 0.5$, Two-Level Non-Homogeneous Earth Systematically Distributed so that each Half of Array is over Different Earth, $\phi = 0°$)

(a) 6 MHz ($\lambda_o = 50$m)

Angle of Incidence, θ (deg)	Elevation Beam Pointing Error, $B_\theta/(BW)_{\theta,3\ dB}$					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.01	0.05	0.03	0.05	0.03	0.16
64	0.01	0.05	0.03	0.05	0.04	0.14
70	0.01	0.05	0.04	0.05	0.05	0.11
74	0.01	0.05	0.04	0.06	0.06	0.10
80	0.02	0.07	0.06	0.08	0.07	0.09
85	0.02	0.06	0.08	0.10	0.10	0.09
90	0	0	0	0	0	0

(b) 30 MHz ($\lambda_o = 10$m)

Angle of Incidence, θ (deg)	Elevation Beam Pointing Error, $B_\theta/(BW)_{\theta,3\ dB}$					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	−0.001	0.16	0.001	0.07	0.00	−0.22
64	−0.001	0.15	0.01	0.06	0.01	−0.22
70	−0.002	0.05	0.02	0.02	0.01	0.21
74	−0.002	0.10	0.02	0.11	0.02	0.20
80	0.004	0.14	0.02	0.07	0.02	0.20
85	0.02	0.13	0.01	0.16	0.03	0.29
90	0	0	0	0	0	0

SECTION 4

SUMMARY AND CONCLUSIONS

The electric field, at each element of a ground-based HF receiving array, is the sum of a direct field and an indirect (multipath) field. For elements with sufficiently-small ground planes, the indirect field is reflected not from the ground plane but from the earth in proximity to that element. The indirect field, relative to the direct field, is the product of the earth Fresnel reflection coefficient and a path-length phase delay that is proportional to the height of the element above the earth. If the earth beneath the array is not homogeneous, then the argument of the total electric field at each element (after allowance for the true phase advance of the direct field at each element) is not uniform from element to element. The non-uniform argument causes an array rms phase error. The non-uniform argument also causes beam pointing errors when the non-homogenous earth is systematically-distributed.

The earth Fresnel reflection coefficients and the arguments of the total field, for a vertically-polarized Hertzian dipole at height h above the earth, are tabulated in this paper for CCIR 527-1 classifications of earth and $h/\lambda_0 = 0$, 0.054, and 0.270. The normalized heights $h/\lambda_0 = 0.054$ and 0.270 correspond to the midpoint at 6 MHz and 30 MHz, respectively, of a 5.4 m length vertical monopole. The RMS phase errors and beam pointing errors are modeled for arbitrary distributions of non-homogeneous earth.

Numerical results are presented for randomly-distributed and systematically-distributed earth non-homogeneities for cases where one-half of the array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. The combinations of earth types that are considered are very dry ground/medium dry ground, medium dry ground/wet ground, and very dry ground/wet ground.

The RMS phase error is an increasing monotonic function of h/λ_o (for modest values of h/λ_o). The RMS phase error is generally a non-monotonic function of the angle of incidence θ.

The maximum expected values of the RMS phase errors at the best diffraction focus of the array for the cases examined, are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The RMS phase error is less for systematically-distributed non-homogeneities because the linear phase error caused by beam pointing errors has been subtracted from the RMS phase error. The maximum expected values of the beam pointing error (in elevation and in azimuth) are 0 and 0.3 beamwidths for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum RMS phase errors and beam pointing errors occur for very dry ground/wet ground, $\theta = 85$ degrees, $h/\lambda_o = 0.270$.

The above numerical results suggest that the influence of non-homogeneous earth is appreciable, but not significant, on the performance of HF receiving arrays with electrically-small ground planes that do not require very low sidelobes.

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APPENDIX

BEAM POINTING ERRORS CAUSED BY A NON-HOMOGENEOUS EARTH

Consider a plane wave that is incident from the true direction (θ, ϕ) at the kth element of a large array of m randomly distributed elements within a circular area of radius r_A. The elements are in proximity to non-homogeneous earth whose non-homogeneities are not necessarily randomly distributed over the circular area. A systematic distribution of non-homogeneities causes beam pointing errors B_θ, B_ϕ that in turn cause a linear phase shift. The beam pointing errors reduce the tracking accuracy of the array. The linear phase shift causes a mean phase-squared error α_B^2 which should be subtracted from the array mean phase-squared error computed at the diffraction focus of the array. Expressions for the beam pointing errors B_θ, B_ϕ and RMS phase error α_B are derived in this appendix.

The true phase advance $\delta_{true,k}$, of the direct field at the kth element, relative to that of an element at the center of the array, is given by reference 1.

$$\delta_{true,k} = (2\pi / \lambda_o)(x_k \sin\theta\cos\phi + y_k \sin\theta\sin\phi) = (2\pi / \lambda_o)(x_k u + y_k v) \qquad \text{(A-1)}$$

where

x_k, y_k = coordinates of the kth element along the x and y axes, respectively, with the origin at the center of the array.

u = $\sin\theta\cos\phi$

v = $\sin\theta\sin\phi$

The measured phase delay δ_k at the kth element, in the presence of an indirect field reflected from the earth, is given by

$$\delta_k = \delta_{true,k} + (\alpha_k - \overline{\alpha}) \tag{A-2}$$

where α_k, $\overline{\alpha}$ are given by equations (2-9) and (2-10), respectively.

The estimates $\hat{u} = \sin\theta\cos\phi$, $\hat{v} = \sin\theta\sin\phi$ may be determined from the least squares algorithm given by

$$\text{minimize with respect to } \hat{u},\hat{v}: \quad \sum_{k=1}^{m}\left[\delta_k - (2\pi/\lambda_o)(x_k\hat{u} + y_k\hat{v})\right]^2 \tag{A-3}$$

The minimization in equation (A-3) is executed by

$$\frac{\partial}{\partial\hat{u}}\sum_{k=1}^{m}\left[\delta_k - (2\pi/\lambda_o)(x_k\hat{u} + y_k\hat{v})\right]^2 = 0 \tag{A-4}$$

$$\frac{\partial}{\partial\hat{v}}\sum_{k=1}^{m}\left[\delta_k - (2\pi/\lambda_o)(x_k\hat{u} + y_k\hat{v})\right]^2 = 0 \tag{A-5}$$

Equations (A-4) and (A-5) reduce to

$$\sum_{k=1}^{m}\left[\delta_k x_k - (2\pi/\lambda_o)x_k^2\,\hat{u} - (2\pi/\lambda_o)x_k y_k\hat{v}\right] = 0 \tag{A-6}$$

$$\sum_{k=1}^{m}\left[\delta_k x_k - (2\pi/\lambda_o)x_k\, y_k\,\hat{u} - (2\pi/\lambda_o)y_k^2\hat{v}\right] = 0 \tag{A-7}$$

The projections x_k and y_k are uncorrelated (because the element locations are randomly distributed). Accordingly,

$$\sum_{k=1}^{m} x_k y_R \approx 0, \text{ elements randomly distributed} \tag{A-8}$$

Substituting equation (A-8) into equations (A-6) and (A-7),

$$\hat{u} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^{m} x_k \delta_k}{\sum_{k=1}^{m} x_k^2} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^{m} x_k[(2\pi/\lambda_o)x_k u + (\alpha_k - \overline{\alpha})]}{\sum_{k=1}^{m} x_k^2}$$

$$= u + \frac{\lambda_o}{2\pi r_A} \frac{\sum_{k=1}^{m} (x_k/r_A)(\alpha_k - \overline{\alpha})}{\sum_{k=1}^{m} (x_k/r_A)^2}; \; x_k, \; y_k \text{ are uncorrelated} \tag{A-9}$$

$$\hat{v} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^{m} y_k \delta_k}{\sum_{k=1}^{m} y_k^2} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^{m} y_k[(2\pi/\lambda_o)x_k v + (\alpha_k - \overline{\alpha})]}{\sum_{k=1}^{m} y_k^2}$$

$$= v + \frac{\lambda_o}{2\pi r_A} \frac{\sum_{k=1}^{m} (y_k/r_A)(\alpha_k - \overline{\alpha})}{\sum_{k=1}^{m} (y_k/r_A)^2}; \; x_k, \; y_k \text{ are uncorrelated} \tag{A-10}$$

It should be noted that $\hat{u} = u$, $\hat{v} = v$ for the cases of homogeneous earth $(\alpha_k = \overline{\alpha})$ and non-homogeneous earth with randomly-distributed elements and non-homogeneities $(\sum x_k(\alpha_k - \overline{\alpha}) = \sum y_k(\alpha_k - \overline{\alpha}) = 0)$. However, for non-homogeneous earth with systematically-distributed elements or non-homogeneities $(\sum x_k(\alpha_k - \overline{\alpha}) = \sum y_k(\alpha_k - \overline{\alpha}) \neq 0)$,

then $\hat{u} \neq u$, $\hat{v} \neq v$.

The RMS phase error α_B contributed by beam pointing errors is given by

$$\alpha_B = (1/m)\sum_{k=1}^{m}[(2\pi r_A/\lambda_o)^2[(x_k/r_A)(\hat{u}-u)+(y_k/r_A)(\hat{v}-v)]^2 \qquad \text{(A-11)}$$

where $\hat{u} = u$, $\hat{v} = v$ are given by equations (A-9) and (A-10), respectively.

Since x_k and y_k have been assumed to be uncorrelated, equation (A-11) reduces to

$$\alpha_B = (2\pi r_A/\lambda_o)[(1/m)(\hat{u}-u)^2\sum_{k=1}^{m}(x_k/r_A)^2 + (1/m)(\hat{v}-v)^2\sum_{k=1}^{m}(y_k/r_A)^2]^{1/2},$$

$$x_k, y_k \text{ are uncorrelated} \qquad \text{(A-12)}$$

Substituting equations (A-9) and (A-10) into equation (A-12),

$$\alpha_B = \left\{\frac{(1/m)[\sum_{k=1}^{m}(x_k/r_A)(\alpha_k-\overline{\alpha})]^2}{\sum_{k=1}^{m}(x_k/r_A)^2} + \frac{(1/m)[\sum_{k=1}^{m}(y_k/r_A)(\alpha_k-\overline{\alpha})]^2}{\sum_{k=1}^{m}(x_k/r_A)^2}\right\}^{1/2}, \qquad \text{(A-13)}$$

$$x_k, y_k \text{ are uncorrelated}$$

The beam pointing errors B_θ, B_ϕ in the elevation and azimuthal directions, respectively, are defined as

$$B_\theta = \theta_{apparent} - \theta \qquad \text{(A-14)}$$

$$B_\phi = \phi_{apparent} - \phi \qquad \text{(A-15)}$$

where $(\theta_{apparent}, \phi_{apparent})$ is the apparent beam direction when the beam has a true direction (θ,ϕ). The beam pointing errors B_θ, B_ϕ are related to $\hat{u}, \hat{v}$ by

$$\hat{u} = \widehat{\sin\theta\cos\phi} = \widehat{\sin(\theta_{apparent} - B_\theta)\cos(\phi_{apparent} - B_\phi)}$$

$$= \text{estimate } [(\sin\theta_{apparent}\cos B_\theta - \sin B_\theta \cos\theta_{apparent})(\cos\phi + B_\phi \sin\phi$$

$$+ \sin\phi_{apparent} \sin B_\phi)] \approx (\sin\theta - B_\theta\cos\theta)(\cos\phi - B_\phi \sin\phi)]$$

$$\approx \sin\theta\cos\phi - B_\theta\cos\theta\cos\phi + B_\phi\sin\theta\sin\phi,\ B_\theta << 1\,rad, B_\phi << 1\,rad \quad \text{(A-16)}$$

$$\hat{v} = \widehat{\sin\theta\cos\phi} = \widehat{\sin(\theta_{apparent} - B_\theta)\sin(\phi_{apparent} - B_\phi)}$$

$$= \text{estimate } [(\sin\theta_{apparent}\cos B_\theta - \sin B_\theta \cos\theta_{apparent})(\sin\phi_{apparent}\cos B_\phi$$

$$- \cos\phi_{apparent}\sin B_\phi)] = (\sin\theta - B_\theta\cos\theta)(\sin\phi - B_\phi\cos\phi)]$$

$$\approx \sin\theta\sin\phi - B_\theta\cos\theta\cos\phi - B_\phi\sin\theta\cos\phi,\ B_\theta << 1\,rad, B_\phi << 1\,rad \quad \text{(A-17)}$$

Rearranging terms,

$$\hat{u} - u = -(\cos\theta\cos\phi)B_\theta + (\sin\theta\sin\phi)\,B_\phi \quad \text{(A-18)}$$

$$\hat{v} - v = -(\cos\theta\sin\phi)B_\theta + (\sin\theta\cos\phi)\,B_\phi \quad \text{(A-19)}$$

Denoting $\Delta = \begin{vmatrix} -\cos\theta\cos\phi & \sin\theta\sin\phi \\ -\cos\theta\sin\phi & -\sin\theta\cos\phi \end{vmatrix} = \cos\theta\sin\theta,$

$$B_\theta = \frac{1}{\Delta}\begin{vmatrix} \hat{u} - u & \sin\theta\sin\phi \\ \hat{v} - v & -\sin\theta\cos\phi \end{vmatrix}$$

$$= (1/\Delta)[(\hat{u}-u)(-\sin\theta\cos\phi) - (\hat{v}-v)(\sin\theta\sin\phi)]$$

$$= -(1/\cos\theta)[(\hat{u}-u)\cos\phi + (\hat{v}-v)\sin\phi], \quad B_\theta << 1\ rad \tag{A-20}$$

$$B_\phi = \frac{1}{\Delta}\begin{vmatrix} -\cos\theta\cos\phi & \hat{u}-u \\ -\cos\theta\sin\phi & \hat{v}-v \end{vmatrix}$$

$$= (1/\Delta)[(\hat{v}-v)(-\cos\theta\cos\phi) - (\hat{u}-u)(\cos\theta\sin\phi)]$$

$$= (1/\sin\theta)[(\hat{u}-u)\sin\phi + (\hat{v}-v)\cos\phi], \quad B_\phi << 1\ rad \tag{A-21}$$

where $\hat{u}-u$, $\hat{v}-v$ are given by equations (A-9) and (A-10), respectively.

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